

A New Approach for Solving Inverse Scattering Problems with Overset Grid Generation Method

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Abstract

This paper presents a new approach of Forward-Backward Time-Stepping (FBTS) utilizing Finite-Difference Time-Domain (FDTD) method with Overset Grid Generation (OGG) method to solve the inverse scattering problems for electromagnetic (EM) waves. The proposed FDTD method is combined with OGG method to reduce the geometrically complex problem to a simple set of grids. The grids can be modified easily without the need to regenerate the grid system, thus, it provide an efficient approach to integrate with the FBTS technique. Here, the characteristics of the EM waves are analyzed. For the research mentioned in this paper, the 'measured' signals are syntactic data generated by FDTD simulations. While the 'simulated' signals are the calculated data. The accuracy of the proposed approach is validated. Good agreements are obtained between simulation data and measured data. The proposed approach has the potential to provide useful quantitative information of the unknown object particularly for shape reconstruction, object detection and others.

Keywords: finite-difference time-domain, forward-backward time-stepping, inverse scattering problems, overset grid generation method

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1. Introduction

Microwave inverse scattering technique is generally used to determine the location, shape and dielectric properties of unknown objects that are scattered by the objects [1]. The original geometrical features can be reconstructed from the scattered data received by the antennas and by numerically time reversing the scattering process. This technique is generally used for the reconstruction of early breast cancer due to its non-destructive effect on healthy tissue [2], military radar imaging [3], and tumour detection [4] and through the wall imaging [5-6]. In general, the inverse scattering techniques are developed in frequency-domain and time-domain for the microwave imaging [7-9]. The single frequency-domain scattering data is usually used in most of the microwave inverse scattering techniques to investigate the inverse problem [10-12]. However, frequency-domain scattering data is often ill-posed due to the nonlinearity and limited measurement parameters available enforced by the problem geometry [13]. In contrast, time-domain has the potential to reconstruct the dielectric properties more accurately [14]. It is therefore imperative to investigate different approaches to decrease the level of ill-conditioning inherent in the inverse problem.

The Forward Backward Time Stepping (FBTS) technique using broadband microwave signals is proposed to formulate the inverse scattering techniques in time-domain. This technique is an alternative approach to microwave imaging [15]. It is a nonlinear inverse scattering computation formulated in the time-domain utilizing Finite-Difference Time-Domain (FDTD) method. The FDTD method, original proposed by Kane Yee [16], proved to be a simple and efficient tool in solving Maxwell's equations. Generally, it is used to improve the detection and reconstruction of the objects [17-19]. The FDTD method contains more information compared to a single-frequency scattering data which would lead to improvements in detection rates. However, there are two major drawbacks to a classical FDTD method [20]. The first one is related to a situation when a two-scale problem occurs. This situation can be caused by a presence of object which is much smaller than the size of the problem. Here, the FDTD need to refine the computational domain globally to solve the problem. However, this method will

increase memory consumption and CPU time [21]. The second drawback is its efficiency with respect to curved boundaries. This method was formulated on tensor product grids and the only way to a curve object is by staircase approximation. Several sub-gridding techniques have been reported in the literature to overcome this limitation by solving the problem separately. First solve the problem in the whole domain on a coarse grid, then part of the FDTD grid is replaced with a finer grid called the sub-grid to solve the sub-problem on finer grid and combine the results [22-25]. With such techniques, it will reduce the overall computational cost but the fine region is restrictive by the Courant-Friedrichs-Lewy (CFL) stability condition [26]. In recent years, many researchers had applied the FDTD method for the inverse scattering technique but all of the above researchers only used a single grid FDTD. To the best of our knowledge the study about the sub-gridding FDTD has not been yet reported with the application of microwave inverse scattering technique.

This paper presents a new computational sub-gridding scheme combining the advantages of FDTD method and Overset Grid Generation (OGG) method is considered to solve the inverse scattering problems for EM waves [27-29]. A system of relatively simple meshes which consists of a static main mesh and static sub-mesh are used for the proposed grid method. These meshes are overlapped on each other in an arbitrary manner to form a single grid [30]. It will reduce the geometrically complex problem to a simple set of grids. Here, a new algorithm will be formulated by integrating the FBTS technique utilizing FDTD method with OGG method for a finer grid. First, the analysis is carried out for direct problem with empty grids and in stationary case for transverse magnetic (TM) mode for two-dimensional (2-D) cases. Several EM pulses are launched to illuminate the unknown scatterers. The characteristics of scattered EM fields between both meshes are measured and analyzed. The measured signals are syntactic data generated by OGG-FDTD simulations. The numerical results are compared to an equivalent simulation. Then, a simple object reconstruction is carried out by using the new approach to solve the inverse problem. This analysis attempts to validate that the proposed approach can be applied to reconstruct unknown objects.

2. Methodology

2.1. Forward-Backward Time-Stepping Technique

The Forward-Backward Time-Stepping (FBTS) technique has the potential to reconstruct images that provide useful quantitative information about the location, shape and the electric properties of the scatter or object. The electric properties include the permittivity (ϵ), permeability (μ) and conductivity (σ). Figure 1 shows a typical configuration of an active microwave tomography setup for FBTS inverse scattering problem in two-dimensional field. The unknown object is assumed to be embedded in a free space. The electrical properties of the unknown object are reconstructed from the transient field data measured at several antennas for each illumination. The errors between the measured and simulated microwave scattering data are compared in the time-domain by using the FBTS technique.

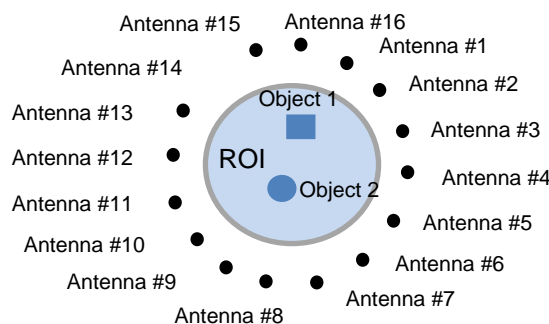


Figure 1. Configuration of FBTS technique in 2-D view

Initiating the FBTS technique, the optimization problem is formulated in the form of cost functional to be minimized [31].

$$Q(p) = \int_0^{cT} \sum_{m=1}^M \sum_{n=1}^N K_{MN}(t) |v_m(p; r_n^r, t) - \tilde{v}_m(r_n^r, t)|^2 d(ct) \quad (1)$$

where p is an electric parameter vector, $v_m(p; r_n^r, t)$ and $\tilde{v}_m(r_n^r, t)$ are the simulated EM fields for an estimated medium parameter vector p and the measured EM fields at the receiving position n due to a pulse radiated by a transmitter m , respectively.

2.2. Overset Grid Generation Method

In this paper, the main mesh covers the entire of the computational domain while the sub-mesh is used to model the unknown object. Each of the grid components in the sub-mesh can be calculated independently from the overlapped main mesh at any overset boundary. Figure 2 illustrates the data transfer between the overlapped meshes by using linear interpolation algorithm.

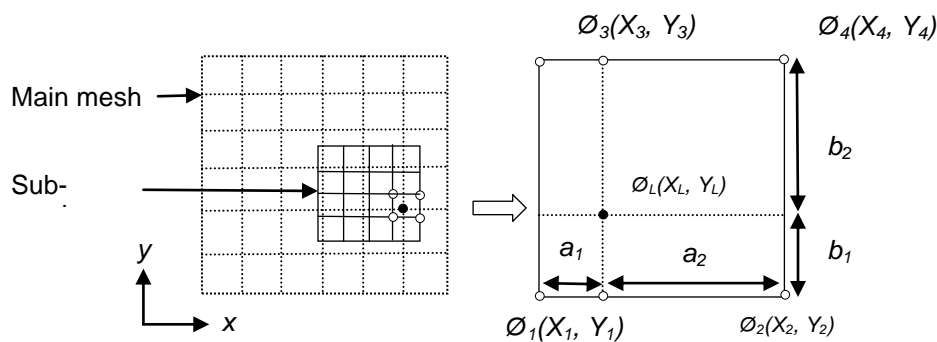


Figure 2. Interpolation model

The unknown value ϕ_L , at point L in the main mesh (dashed line) can be determined from the existing factors $\phi_1, \phi_2, \phi_3, \phi_4$ in the sub-mesh (solid line) as in equation (2) [32]:

$$\phi_L = \frac{(\phi_1 \times a_2 + \phi_2 \times a_1) \times b_2 + (\phi_3 \times a_2 + \phi_4 \times a_1) \times b_1}{(a_1 + a_2)(b_1 + b_2)} \quad (2)$$

The value at the interpolation point is used to transfer inter-grid information and recomputed at each time step.

2.3. Computational Algorithm of FBTS technique utilizing OGG method and FDTD method (OGG-FDTD)

The 2-D FDTD formulations for TM cases are produced by assuming that all partial derivatives of the fields with respect to z -direction are equal to zero. In order to formulate the OGG algorithm into the FDTD algorithm, the flow chart of computing is indicated in Figure 3. The computation domain is separated into two parts to apply the OGG-FDTD algorithm. One part is for the main fields region in FBTS algorithm (*System A*) and the other is for the OGG-FDTD algorithm (*System A'*). The EM fields of the main fields region need to be determined in *System A'* for every time step. The EM fields of the related grids in *System A* are updated with the corresponding calculated EM fields in *System A'* which consists of main mesh and sub-mesh. In *System A'* the position of the sub-mesh is identified. The EM field components on the main mesh are interpolated to the field components on the sub-mesh through Lorentz transformation. The electric field is calculated in both meshes by using the FDTD method. The calculated value of the electric field at the sub-mesh is interpolated back to the main mesh by applying Lorentz transformation. Then, the electric field is updated in the main fields region in *System A* where the half time increment is advanced. The same process is iterated to calculate

the magnetic field. The difference between the EM fields for an estimated medium parameter vector and the measured EM fields are calculated in *System A* by using the FBTS technique in time-domain. In this paper, the time component in main fields region in FBTS algorithm is set as $t = t + \Delta t$. The time step (Δt) is determined by using the Courant-Friedrichs-Lewy (CFL) stability condition. The time component in the OGG-FDTD algorithm is fixed as $t' = t' + \Delta t / 2$, so that the time component on the main mesh can be calculated and interpolated to the sub-mesh. The process continues until the time-stepping is concluded. The proposed procedure is the key idea of the approach to apply the interpolation technique into the time component.

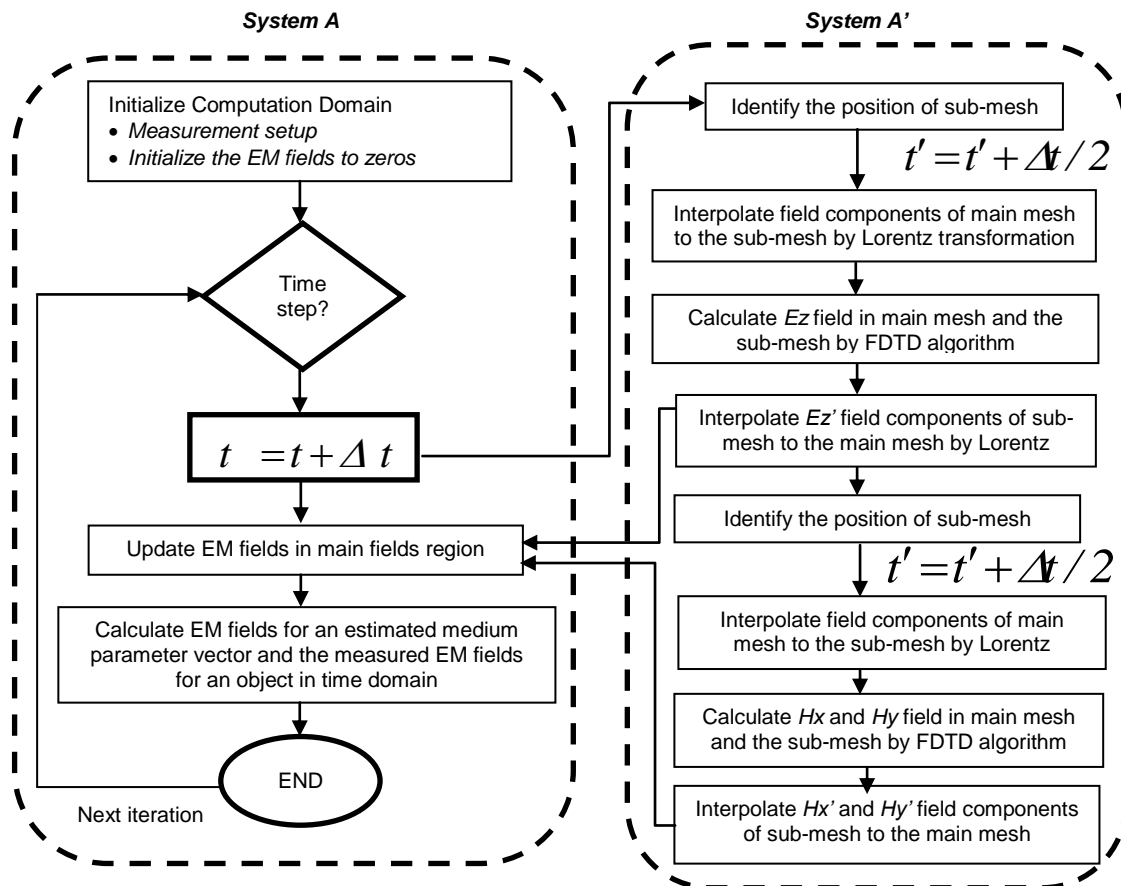


Figure 3. Computational OGG-FDTD algorithm

3. Numerical Model and Simulation Setup

Figure 4 illustrates the numerical model for FBTS and FDTD with OGG method. The main mesh is set to 190×190 mm and the sub-mesh is 50×50 mm. The sub-mesh is located at the center of main mesh. There are 16 antennas are utilized in the analysis. Each antenna will become transmitter sequentially to transmit a pulse into the FDTD lattice environment while the remaining antennas will become receiver to collect the scattered signal from the FDTD lattice environment. Here, the analysis is carried out with empty grids and in stationary case to validate the proposed technique. The entire FDTD lattice environment is set as free space with relative permittivity, $\epsilon_r = 1.0$ and $\sigma = 0.0$. The incident wave source is a sinusoidal modulated Gaussian pulse with a center frequency, $f_c = 2$ GHz is assigned to electric component, E_z in the FDTD lattice.

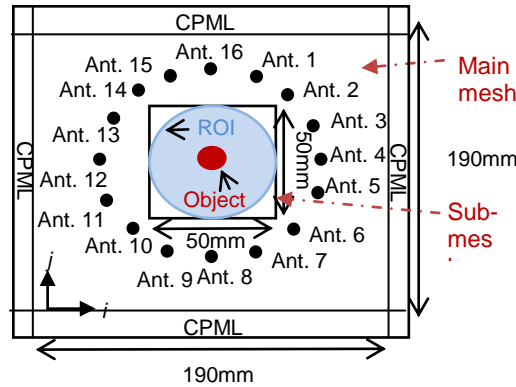


Figure 4. Numerical model

$$E_z \Big|_{Gp}^{n+1/2} = A e^{-\left(\frac{t-t_0}{td}\right)^2} \sin(2\pi f(t-t_0)) \quad (3)$$

The boundary is terminated with the convolutional perfectly matched layer (CPML) with thickness of 15mm at the borders of the FDTD lattice to reduce the reflection from the environment and to reduce the computational domain. The space increment for the main mesh is $\Delta x_m = \Delta y_m = 1.0\text{mm}$, and for the sub-mesh is $\Delta x_s = \Delta y_s = 1.0\text{mm}$. The grid size ratio, R between main mesh and sub-mesh is given by $R = \frac{\Delta x_s}{\Delta x_m}$, where the value of $R=1.0$.

4. Result and Discussion

The analysis is divided into two parts: The first part is for direct problem where EM pulses are launched to the sub-gridding FDTD to illuminate the unknown scatterers and solved based on OGG-FDTD method. The second part is for inverse problem where the shape, size and location of embedded object are reconstructed by the scattered electric field obtained at the receivers.

4.1 Direct Problem – Signal Analysis

The simulation is carried out by using sub-gridding FDTD without an object for signals analysis. In this analysis, two antennas are used as transceiver. For the research mentioned in this paper, the 'measured' signals are syntactic data generated by OGG-FDTD simulations. While the 'simulated' signals are the calculated data. The electrical fields at the receiving antenna is represented as measured signal ($R_{X_{MEAS}}$) and simulated signal ($R_{X_{FWD}}$). The errors between $R_{X_{MEAS}}$ and $R_{X_{FWD}}$ are measured and compared by using the FBTS technique in the time domain. The observed data at the receiving antenna position Rx1 and Rx2 is simulated for single-Tx-single-Rx configuration by utilizing the FDTD method and by utilizing OGG-FDTD method. Figure 5 (a) shows the $R_{X_{FWD}}$ at position Rx1 and Figure 5 (b) shows the $R_{X_{MEAS}}$ at position Rx1. The solid line represents the implementation of FBTS by utilizing the FDTD and the dashed line represents the implementation of FBTS by utilizing the OGG-FDTD method. Figure 5 (c) shows the $R_{X_{FWD}}$ at position Rx2 and Figure 5 (d) shows the $R_{X_{MEAS}}$ at position Rx2. The solid line represents the implementation of FBTS by utilizing the FDTD and the dashed line represents the implementation of FBTS by utilizing the OGG-FDTD method.

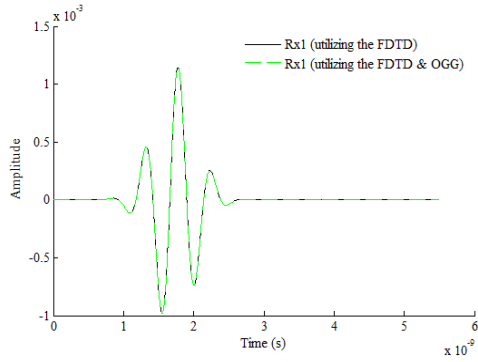


Figure 5 (a). RxFWD at position Rx1

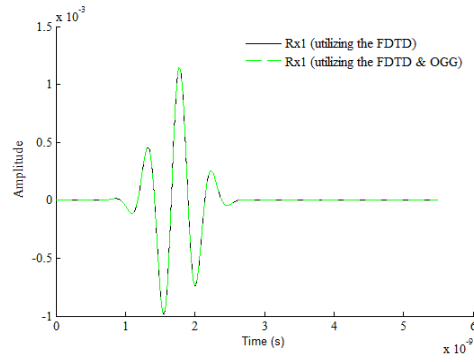


Figure 5 (b). RxMEAS at position Rx1

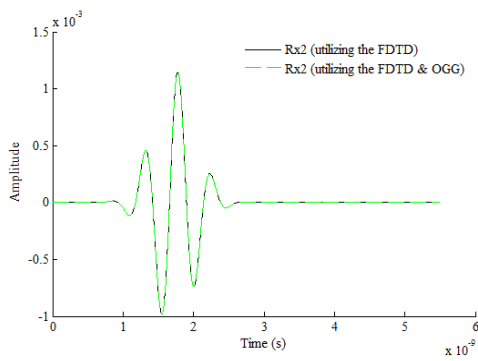


Figure 5 (c). RxFWD at position Rx2

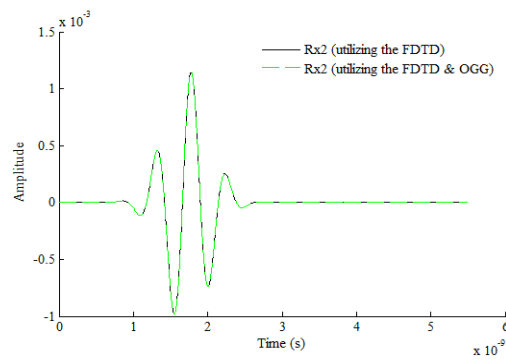


Figure 5 (d). RxMEAS at position Rx2

The error signals between the Rx_{MEAS} and Rx_{FWD} between FBTS by utilizing the FDTD and FBTS by utilizing the OGG-FDTD method are compared by using the mean-squared error (MSE). The MSE between the signals is given by:

$$MSE(x, y) = \frac{1}{N} \sum_{i=1}^N (x_i - y_i)^2 \tag{4}$$

where x_i is the signals in FBTS by utilizing the FDTD method are set as reference signals, y_i is the signals in FBTS by utilizing the OGG-FDTD method and N is the number of signal samples. The comparison of Rx_{MEAS} and Rx_{FWD} is shown in the Table 1.

Antenna Position	Signal	MSE (10^{-17})
Rx1	RxFWD	9.7875
	RxMEAS	9.7875
Rx2	RxFWD	9.8262
	RxMEAS	9.8262

From Table 1, it shows that the signals in FBTS by utilizing OGG-FDTD method gives lower MSE value. This indicates the signals obtained by utilizing OGG-FDTD method produced less error and nearer to the reference signals. It proves that the proposed approach can be applied to reconstruct unknown objects.

4.2. Inverse Problem - Reconstructions of the Object

For the inverse scattering problem, a simple circular object embedded in the region of interest (ROI) is conducted by using the proposed new approach as in Figure 4. The size of the ROI is set with the diameter of 50mm while the size for the object is set to 15mm in diameter. The dielectric properties of ROI is fixed with $\epsilon_r = 9.98$ and $\sigma = 0.18$. The ROI is surrounded by 16 antennas which are used to transmit Gaussian pulse. The embedded object is located at the center of ROI with the dielectric properties is set with $\epsilon_r = 21.45$ and $\sigma = 0.45$. The initial guess for ϵ_r and σ values for the simulation are 13.7 and 0.10 respectively which nearer to the actual profiles. Figure 6 illustrates the actual profiles of the circular model used for the simulation. Figure 7 shows the reconstructed profiles by employed the FBTS technique utilizing FDTD method only. The embedded objects can be detected and reconstructed. Figure 8 shows the preliminary results of the new approach for reconstructions of the object. The reconstructed object has been carried out for 100 iterations. The results shows that the new approach has been successfully detected and reconstructed the object with the MSE value of the reconstructed dielectric properties are 0.14052 for relative permittivity and 0.00016 for conductivity in both techniques. It is the same with the reconstructed object by employed the FBTS technique utilizing FDTD method because the grid size ratio, $R = 1.0$.

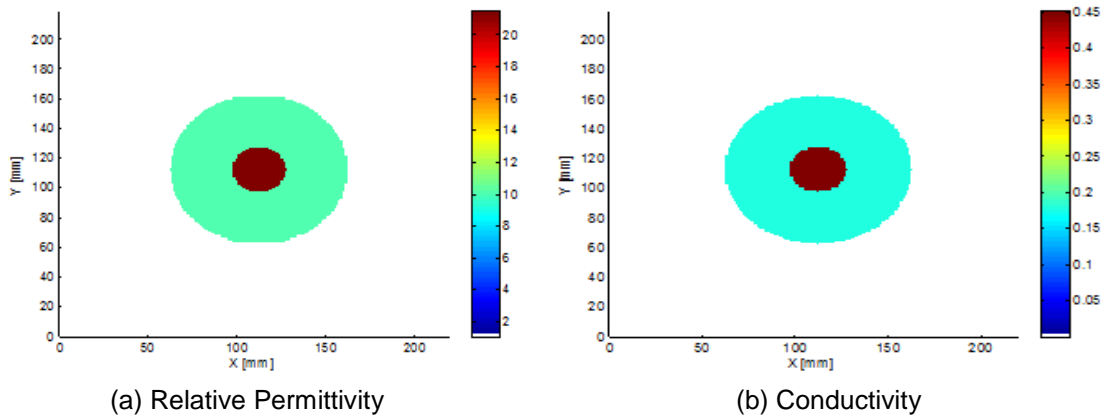


Figure 6. Actual Profiles

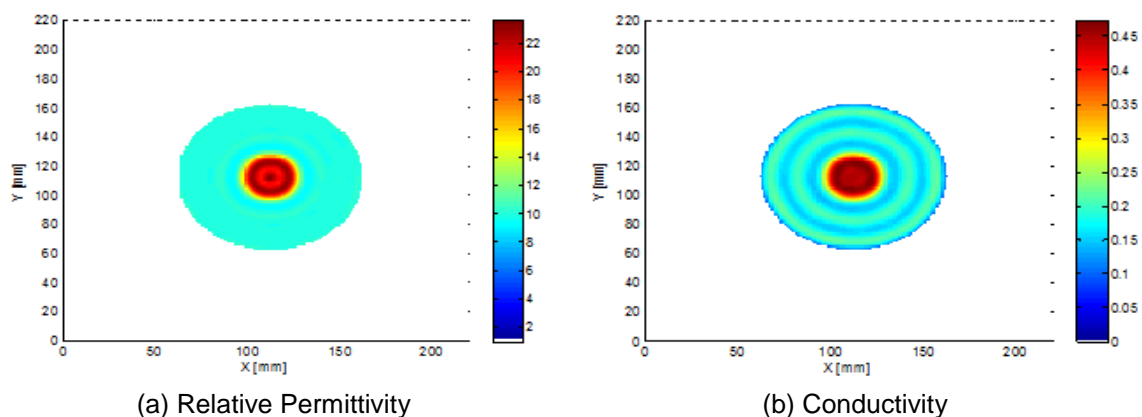


Figure 7. FBTS with FDTD method: Reconstructed Profiles

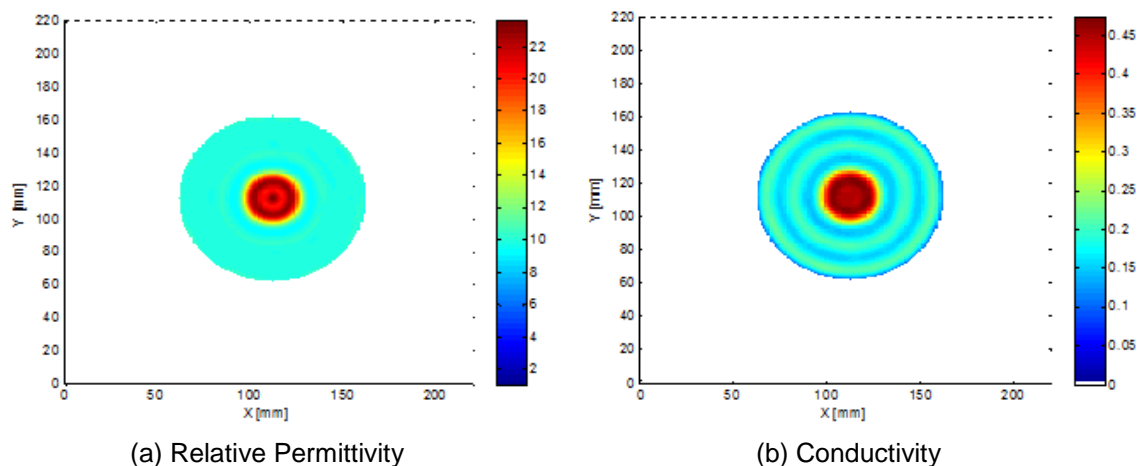


Figure 8. FBTS with OGG-FDTD method: Reconstructed Profiles

5. Conclusions and Future Work

For direct problem, the characteristics of the simulated signal and measured signal for the development of the FBTS algorithm by utilizing OGG-FDTD method are presented. The results show good agreements are obtained with the reference signals. For inverse problem, the proposed approach is proven that has the potential to reconstruct the simple circular object embedded with the quantitative information regarding the shape, size and location of the object. Future work will include the analysis of difference size circular object embedded in free space.

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