

Curtailed-Gaussian and Cosine Functions for Multihop Doppler Spectrum Modeling

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Abstrak

Salah satu karakteristik kanal yang mencirikan kanal wireless adalah spektrum Doppler. Pada diversitas kooperatif, salah satu cabangnya dimungkinkan terdiri dari beberapa relay bergerak sehingga membentuk link multihop, dimana tiap hop-nya memberikan kontribusi pergeseran Doppler. Bila digunakan relay amplify-and-forward (AF), pergeseran Doppler tersebut akan diteruskan dan makin bertambah sampai ujung akhir link. Besarnya pergeseran Doppler mempengaruhi laju perubahan kanal terhadap waktu yang merupakan tantangan dalam desain komunikasi bergerak pita lebar. Oleh karena itu, parameter Doppler sangat penting dan harus diperhatikan dalam disain dan analisa sistem komunikasi bergerak pita lebar. Sayangnya, pernyataan bentuk tertutup dari spektrum Doppler tersebut secara matematis sulit diturunkan karena terdapat fungsi khusus dalam integrasi, yaitu fungsi integral eliptik lengkap. Untuk mengatasi masalah ini, digunakan metode curve-fitting berbasis least-square. Pada proses ini, diusulkan dua fungsi yaitu curtailed-Gaussian dan cosine sebagai fungsi pendekatan. Selanjutnya, dari uji divergensi Kullback-Leiber juga dapat ditunjukkan bahwa kedua fungsi yang diusulkan, yaitu fungsi curtailed-Gaussian dan cosinus mempunyai pendekatan yang baik untuk digunakan sebagai pemodelan spektrum Doppler dari kanal bergerak multihop dengan asumsi bahwa penguatan relay AF dinormalisasi berharga 1 dan semua terminal bergerak diasumsikan berpindah dengan kecepatan yang hampir sama.

Kata kunci: spektrum, Doppler, pemodelan, multihop, bergerak

Abstract

Wireless channels are characterized among others by their Doppler spectrum. In the cooperative diversity, one of diversity branch may consist of several mobile relays forming multihop link which each hop introduced Doppler shift. With employing amplify-and-forward (AF) relays, the Doppler shift keeps accumulating to the end of the link. Doppler shift value affects the time varying channel rate, which is a challenge in broadband mobile communication system. Hence, the Doppler parameter is very important and must be considered in broadband mobile communication system design and analysis. Unfortunately, it is hard to derive the expressions of this Doppler spectrum in a closed form since a special function under integration such as complete elliptic integral exists. To solve this problem, curve-fitting method base on least-square is used. In this process, curtailed-Gaussian and cosine functions are proposed as an approximation function. Then, from Kullback-Leiber divergence test, it is showed that both proposed functions, i.e., curtailed-Gaussian and cosine functions have a good approximation as Doppler spectrum modeling of Multihop mobile channel with all gain relays assumed as 1 and all mobile terminals are assumed move with almost same velocity.

Keywords: Doppler, spectrum, modeling, multihop, mobile

1. Introduction

Multihop transmission with moving relays has attracted many researchers recently [1]-[8], due to its useful applications to achieve cooperative diversity for mobile cellular communication network. Such mobile-to-mobile communication systems differ from the conventional cellular radio systems, where one terminal, i.e., the base station (BS), is stationary, and only the mobile station (MS) is moving. Although the received signal envelope is Rayleigh faded under non-line-of-sight (N-LOS), but the mobility of both the transmitter (Tx) and the receiver (Rx) result in different statistical properties of the wireless channels, as described first by Akki and Haber [9], [10]. Their analysis was extended by Patel et al in [11] by proposing a "double-ring" model, which was developed based on sum-of-

sinusoids (SoS) models for simulating such channels. This channel model was used by Patel and Stuber in [12] to analyze statistical properties of amplify-and-forward relay fading channels. They noticed that the closed form expression for the Doppler spectrum for dual-hop; i.e. a mobile-to-mobile link in cascade with a mobile-to-fixed link, remains an open problem.

In cooperative communications, there are different schemes of relay processing namely non-regenerative amplify-and-forward (AF) and regenerative decode-and-forward (DF) scheme. The AF scheme is widely used for cooperative diversity, because it is easy to implement and yields satisfactory performance [1]. It also has been shown in [1], [13] that the AF scheme has about the same quality as the DF scheme at high SNR in terms of its diversity.

In the cooperative diversity, the cooperation link, which is the link that involves the use of relays, is often modeled as dual-hop, i.e. using a single relay. Whereas in cellular networks, a lot of mobile terminals are available to perform the relaying function and assist other users in delivering the message to remote destination. The existing dual-hop model can be extended to Multihop with all the relays assumed to be mobile. To the best of our knowledge, very little is known about the Doppler spectrum of Multihop mobile-to-mobile link, based upon which a cooperative system can be evaluated.

Our contribution is two-fold: firstly, a new Doppler spectrum models for mobile-to-mobile Multihop channels with AF relays and secondly, a mathematical approximation of the Doppler spectrum model. More specifically, as motivated by the fact that the overall channel consists of mobile-to-mobile channels in cascade and can be modeled as the product of N fading amplitudes, the Doppler spectrum is also propagated and accumulated from one hop to the next up to the destination. Closed-form expressions of the desired Doppler spectrum seem to be intractable and therefore, we use curve-fitting method to the Doppler spectrum curve which was obtained numerically. We propose functions as simple and close as possible to the Doppler spectrum as models.

The remainder of the paper is organized as follows. Section II introduces the Multihop mobile-to-mobile channels based on double-ring scatterers channel model and presents numerical results of Multihop Doppler spectrum models. The curve-fitting formulation of the model is presented in section III. Results and analysis are presented in section IV. Finally, section V provides some conclusions.

2. Proposed Method

In this section, Multihop mobile to mobile fading channel and doppler spectrum are explored.

2.1 Multihop Mobile to-Mobile Fading Channel

The Multihop uplink transmission is depicted in Figure 1 in which a mobile station (MS) source (i.e. node 0) is sending data via MS relays (i.e. node 1 to node $n-1$) to the base station (BS) destination (i.e. node n). The Multihop transmission might constitute a diversity branch in cooperative communication systems that consist of relaying chain process.

For the scenario of n -hop transmissions, S, R_1, \dots, R_{n-1} , and D, denote a source, relays, and a destination, respectively. The transmitted signal, $s(t)$, after travelling through n -hop channel and amplify $(n-1)$ times by $(n-1)$ will received as $r_n(t)$. Analogy with received signal $r_2(t)$, of two-hop as derived in [12], the received signal, $r_n(t)$, of n -hops can be derived as follow:

$$\begin{aligned}
 r_1(t) &= A(t)h_1(t)s(t) + n_1(t) \\
 r_2(t) &= A(t)h_2(t)r_1(t) + n_2(t) \\
 r_3(t) &= A(t)h_3(t)r_2(t) + n_3(t) \\
 &\vdots \\
 r_n(t) &= A(t)h_n(t)r_{n-1}(t) + n_n(t) \\
 r_n(t) &= (A(t))^{n-1} (\prod_{k=1}^n h_k(t))s(t) + \sum_{k=1}^{n-1} (A(t))^{n-k} (\prod_{l=k+1}^n h_l(t))z_k(t) + z_n(t) \quad (1)
 \end{aligned}$$

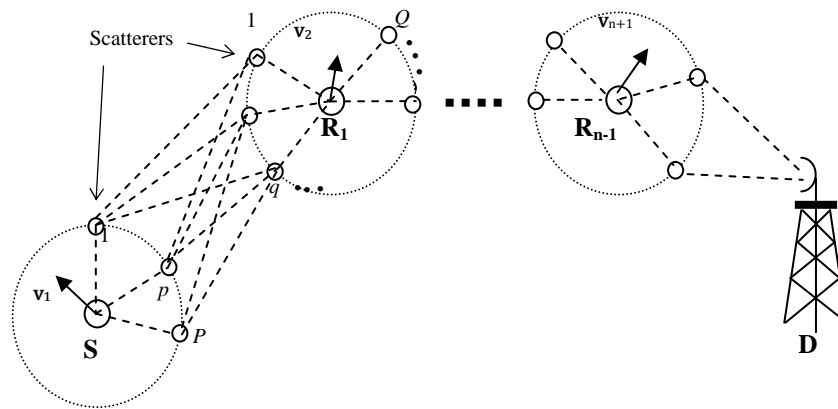


Figure 1. The Multihop mobile-to-mobile fading channels

Where $A(t)$ denotes the relay gain which is assumed to be identical for all relays; $h_k(t)$, the fading factor of the k -th channel that ends at the k -th relay or the BS for $k = n$, $z_k(t)$ the zero-mean additive white Gaussian noise (AWGN) with power spectrum density N_k at node k . All noises introduced by relays and destination have identical statistic properties. For further discussions and simplicity, the time index is dropped. All amplification factors are, $A(t)$, considered fixed and normalized as 1.

For the mobile-to-mobile fading channel, the channel models derived by Patel in [11] is used and is given as:

$$h_{MM}(t) = \sqrt{\frac{2}{PQ}} \sum_{p=1}^P \sum_{q=1}^Q \exp(j(2\pi(f_1 \cos(\alpha_p)t + f_2 \cos(\beta_q)t) + \theta_{p,q})) \quad (2)$$

$f_1 = v_1/\lambda_1$ and $f_2 = v_2/\lambda_2$ are the maximum Doppler shifts induced by the motion of the source MS with speed v_1 and carrier wavelength of λ_1 and the relay MS with speed v_2 and carrier wavelength of λ_2 . The index p refers to the p -th propagation path from the Tx to the p -th of the P scatterers located on the Tx end ring, while the index q refers to the q -th paths from the q -th of the Q scatterers on the Rx end ring to the Rx. The phases $\theta_{p,q} \sim U(-\pi, \pi]$ are independent for all p, q pairs. α_p and β_q are angles of departure and arrival to/from the scatterers at Tx and Rx, respectively. For the Multihop link, each mobile-to-mobile hop is modeled by this channel model which is essentially the same model for all hops except the last one. The last hop is a mobile-to-fixed channel, for which a single-ring scatterers channel model derived in [14] is used. This channel model is given as follow:

$$h_{MF}(t) = \sqrt{\frac{2}{P}} \sum_{p=1}^P \exp(j(2\pi f_1 \cos(\alpha_p)t + \theta_p)) \quad (3)$$

The dynamic behavior of the relay channels can be described by the time autocorrelation function. For dual-hop transmission with fixed gain relay, the overall impulse response is $h = h_1 h_2$, where h_1 is a mobile-to-mobile channel and h_2 is a mobile-to-fixed channel. The auto-correlation function can be derived as follow,

$$\begin{aligned} R_{hh}(\tau) &= \frac{1}{2} E[h_1(t)h_2(t)h_1^*(t+\tau)h_2^*(t+\tau)] \\ &= 2R_{h_1 h_1}(\tau)R_{h_2 h_2}(\tau) \\ &= 2J_0(2\pi f_1 \tau)J_0(2\pi f_2 \tau)J_0(2\pi f_3 \tau) \end{aligned} \quad (4)$$

Where $E[.]$ is the statistical expectation operator, $*$ denotes the complex conjugate operator, $J_0(.)$ is the zeroth order Bessel function of the first kind. In the second line we exploit the independence of h_1 and h_2 and substitute both autocorrelation value of h_1 and h_2 with autocorrelation function of mobile-to-mobile channel derived in [11] and autocorrelation function of mobile-to-fixed derived in [14], respectively. Hence the autocorrelation function consists of the product of three Bessel functions with different maximum Doppler shift i.e., f_1 as source maximum Doppler shift, f_2 and f_3 as relay maximum Doppler shift at received and transmitted state, respectively.

For n -hop link, from (1) the overall response impulses are $h_1 \cdot h_2 \cdot \dots \cdot h_n$, with h_m , $m = 1, \dots, n-1$, being mobile-to-mobile channels, meanwhile h_n is a mobile-to-fixed channel. Analogous to the dual-hop, the auto-correlation function can be derived as the product of n Bessel functions with possibly different maximum Doppler shifts and a scale factor of 2^{n-1} .

$$R_{hh}(\tau) = 2^{n-1} (\prod_{m=1}^{2n-1} J_0(2\pi f_m \tau)) \quad (5)$$

Following the expression for rms Doppler spread, β_d , as defined in [15], it can be easily shown that for n -hop,

$$B_d = \sqrt{\frac{\sum_{m=1}^{2n-1} f_m^2}{2}} \quad (6)$$

2.2 Multihop Mobile to-Mobile Doppler Spectrum

Motion in the wireless environment causes transmitted signals to experience Doppler shifts as a function of the angle between the signal path and the direction of motion. Signals arrive at a receiver from moving mobile terminals through reflections from scatterers at a variety of angles. The superposition of these multipath components having various Doppler shifts and received powers appears as the Doppler spread at the receiver. The exact form of the Doppler spectrum depends on the wireless environment and is the Fourier transform of auto-correlation function of the channel response. The well-known U-shaped Doppler spectrum of cellular channels [9] with normalized channel power gain is given by,

$$S_{MF}(f) = \frac{1}{\pi^2 \sqrt{f_1^2 - f^2}} \quad (7)$$

The Doppler spectrum for mobile-to-mobile channels is obtained as function of the complete elliptic integral of the first kind $K(.)$ given by [11],

$$S_{MM}(f) = \frac{2}{\pi^2 \sqrt{f_1 d}} K\left(\frac{1+d}{2\sqrt{d}} \sqrt{1 - \left(\frac{f}{(1+d)f_1}\right)^2}\right) \quad (8)$$

where $d = f_2/f_1$ is defined as the Doppler ratio for single-hop. Hence for n -hop that consists of $(n-1)$ -hop mobile-to-mobile fading channels and 1-hop of mobile-to-fixed fading channel, the closed-form of Doppler spectrum may be obtained by convolution operation of $(n-1)$ Doppler Spectrum in (8) and a Doppler spectrum in (7) as mentioned above or by taking the Fourier Transform of (5), i.e. products of $(2n-1)$ Bessel functions. Unfortunately, finding a closed form expression for the Doppler spectrum using either of these two operations is rigorous since they both have a special function under integration such as complete elliptic integral. To overcome this difficulty, we use numerical method to find the shape of Doppler spectrum. For simplicity, the Doppler spectrum is expressed as functions of the source' maximum Doppler shift f_1 and the Doppler ratio d_m defined as,

$$d_m = \frac{f_{m+1}}{f_1}, \quad m = 1, \dots, N; \quad N = 2n - 2 \quad (9)$$

Where f_N is the maximum Doppler shift of the last hop. Therefore, for fixed f_1 , different spectrum is obtained by varying the Doppler ratio d_m . For dual-hop channels, the Doppler spectrum has several shapes with one or two discontinuity points and is more concentrated near zeros when the two mobile stations move with almost identical velocity vectors [12].

For n -hop link, the Doppler spectrum is obtained by taking the Fourier Transform of the time's autocorrelation function that consists of the product of $(2n - 1)$ Bessel functions. If d_m has a value near one (such as, $d_m = 1 \pm 0.2$), it means all mobile stations move with almost identical velocity. The Doppler spectrum has a shape as seen in Figure 2. This figure shows that the Doppler spectrum shape does not change much and is still concentrated near zero as in dual-hop (in this figure, all Doppler ratio is set to 1 and $f_1 = 70$ Hz) and the maximum Doppler shift, f_{max} , is defined as:

$$f_{max} = f_1 + \sum_{m=1}^{2n-2} d_m f_1 \quad (10)$$

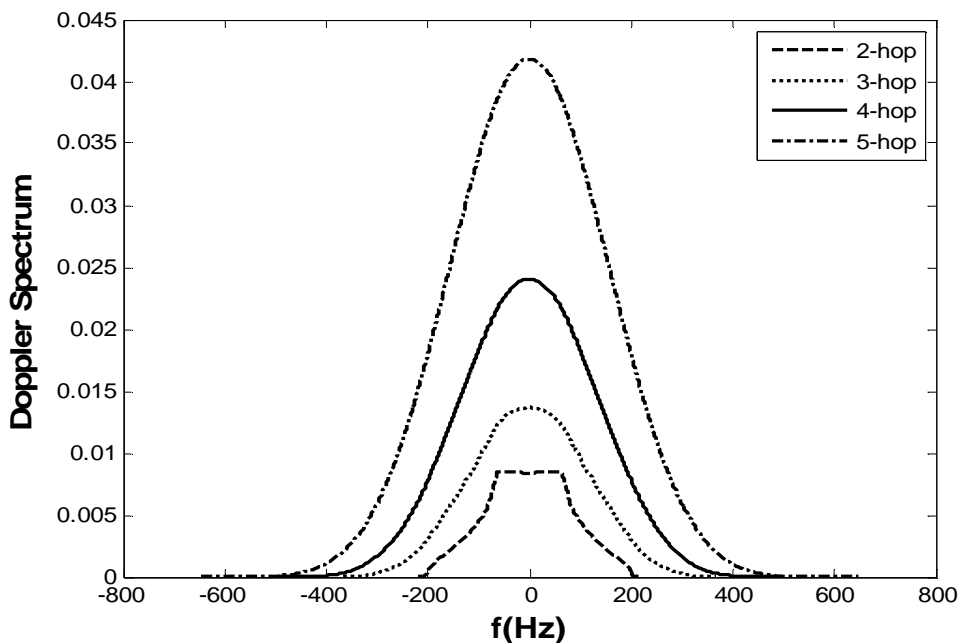


Figure 2. Doppler spectrum for Multihop mobile-to-mobile channel with double-ring scatterers

3. Research Method

Curve-fitting method is applied to the Doppler spectrum of Multihop mobile-to-mobile channel for getting the approximation models. Several functions that best fit the normalized Doppler spectrum in the least-square sense are used. Based on figure 2, the Doppler spectrum shape can be approximated with a curtailed-Gaussian and a cosine functions as the Multihop number increased. So with this reason, we choose these functions to express the models.

Another reason that supports this idea (choosing the curtailed-Gaussian function for the approximation) is described as follow:

Given the product of M Bessel functions:

$$x(\tau) = \prod_{m=1}^M J_0(2\pi f_m \tau) \quad (11)$$

Then its Fourier Transform can be expressed as:

$$X(f) = S_1(f) * S_2(f) * \dots * S_M(f) \quad (12)$$

$S_M(f)$ is as defined in (7) and (8) and $*$ denotes convolution operator. This form resembles that of pdf (probability density function) of a random variable, which is the sum of M independent random variables of the same shape but with different parameters. Consequently, by invoking the Central Limit Theorem [16], the Fourier transform of the product (12) approaches a curtailed-Gaussian function. This is true when M is sufficiently large, i.e. the number of hops with moving relays is large. Cosine function also proposed as alternative models because the Doppler spectrum curve is nonlinear and the shape also close to half cycle cosine function, $\cos(f)$, that has maximum value at $f = 0$ in the interval $-\pi/2 \leq f \leq \pi/2$.

Curve fitting with curtailed-Gaussian and cosine functions is done by choosing several coefficients associate with Doppler parameter such as maximum Doppler shift and number of hops. Curve-fitting procedures can describe as follow,

1. First, the basic equation is set up for each approximation function,

Curtailed-Gaussian function:

$$S_G(f) = \frac{G\alpha}{\sqrt{2\pi}a} \exp\left(-\frac{(f/f_{max})^2}{2a^2}\right); |f| \leq f_{max} \quad (13)$$

Cosine function:

$$S_C(f) = G\beta(\cos(2\pi bf/f_{max}))^{19.5}; |f| \leq f_{max} \quad (14)$$

Each function has several independent variables, consists of G, α, a, f and f_{max} for curtailed-Gaussian and G, β, b, f and f_{max} for cosine. f_{max} is related to f_1 and d_m through equation (10). f_1 and d_m are defined first with several value of f_1 from 10 to 600 Hz (commonly, Doppler shift in cellular system is in this range) and d_m set as random with $d_m \sim N(\mu = 1, \sigma^2 = 0.1)$.

2. Doppler spectrum is computed and plotted based on numerical computation, for each n .
3. Each of curtailed-Gaussian and cosine function in step 1 is fitted to the Doppler spectrum in step 2 based on least squares methods. Matlab toolbox program nlinfit is used in fitting process.
4. From step 3, coefficients in (13) and (14) are obtained for several n and f_1 values.
5. Each coefficient in step 4 (i.e., G, α, β) is plotted as function of f_1 for different n .
6. Approximation functions with independent variable f_1 are created for each coefficient curve in step 5. Each coefficient is related to several coefficients (i.e., $a_1, a_2, a, b_1, b_2, b, g_1$ and g_2) through following equations,

$$G = g_2 + \frac{g_1}{f_1}; \alpha = \alpha_2 + \frac{1}{a_1 f_1}; \beta = b_2 + \frac{b_1}{f_1} \quad (15)$$

7. Step 5 – 6 are repeated for ($a_1, a_2, a, b_1, b_2, b, g_1$ and g_2) coefficient curve as function of n .
8. From step7, again by using curve-fitting method, every coefficient: $a_1, a_2, a, b_1, b_2, b, g_1$ and g_2 are presented as function of n as given in table I.

4. Results and Discussion

In Figure 3, the Doppler spectrum curve obtained from numerical method and from the proposed models (curtailed Gaussian and cosine) for multihop mobile channel seem have a good approximation. For this reason, the Kullback-Leiber divergence (denoted by D_{KL}) is used to validate the models. Here, the D_{KL} is used as a measure of the difference between reference (numerical) Doppler spectrum curve and Doppler spectrum curve obtained from proposed models. From [17] is defined:

$$D_{KL}(V||W) = \sum_i V(i) \ln \frac{V(i)}{W(i)} \quad (16)$$

$V(i)$ and $W(i)$ are Doppler spectrum from the reference (numerical) and the proposed models, respectively. The Kulback-Leiber divergence is always non-negative and equal to zero if and only if $V = W$. So, the closer D_{KL} is to 0, the better is the approximation of the model with the theory. The results of this comparison, presented in Table II, show that the Doppler spectrum of our models provides a good approximation to the numerical Doppler spectrum.

Table 1. Doppler spectrum coefficients of curtailed-Gaussian & cosine model

n	a_1	a_2	a	b_1	b_2	b	g_1	g_2
2	20.4	2.4	0.45	19.1	1.85	0.08	0.26	$6 \cdot 10^{-4}$
3	12.4	2.2	0.33	15.0	2.44	0.11	0.32	$6 \cdot 10^{-4}$

$a_1 = -0.85 + 40/n$	$b_1 = -0.5525 + 26.684/\sqrt{n}$
$a_2 = 2.05$	$b_2 = -0.4 + 1.7\sqrt{n}$
$a = 0.085 + 0.8/n$	$b = -0.0189 + 0.074\sqrt{n}$
$g_1 = (0.00054 + 0.118/n)2^n$	$g_2 = 2.55 \cdot 10^{-5}n^2 + 1.4 \cdot 10^{-5}n^9$

Further investigation on n -Multihop shows that if m out of $n - 1$ mobile relays are not moving (stopping during traveling), than the number of Bessel factors is decreased by $2m$ and the Doppler spectrum width will reduce to $(n - m)$ -hop Doppler spectrum, but the power remains high, which is close to n -Multihop power. If Doppler ratio is assumed as random with normal distribution, $d_m \sim N(\mu = 1, \sigma^2 = 0.1)$, the models still provide a good approximation.

The Doppler spectrum models can be used to analyze system performance with adaptive modulation applications [18], because M-ary modulation with high M has narrow bandwidth and more sensitive to spectral spread due to Doppler.

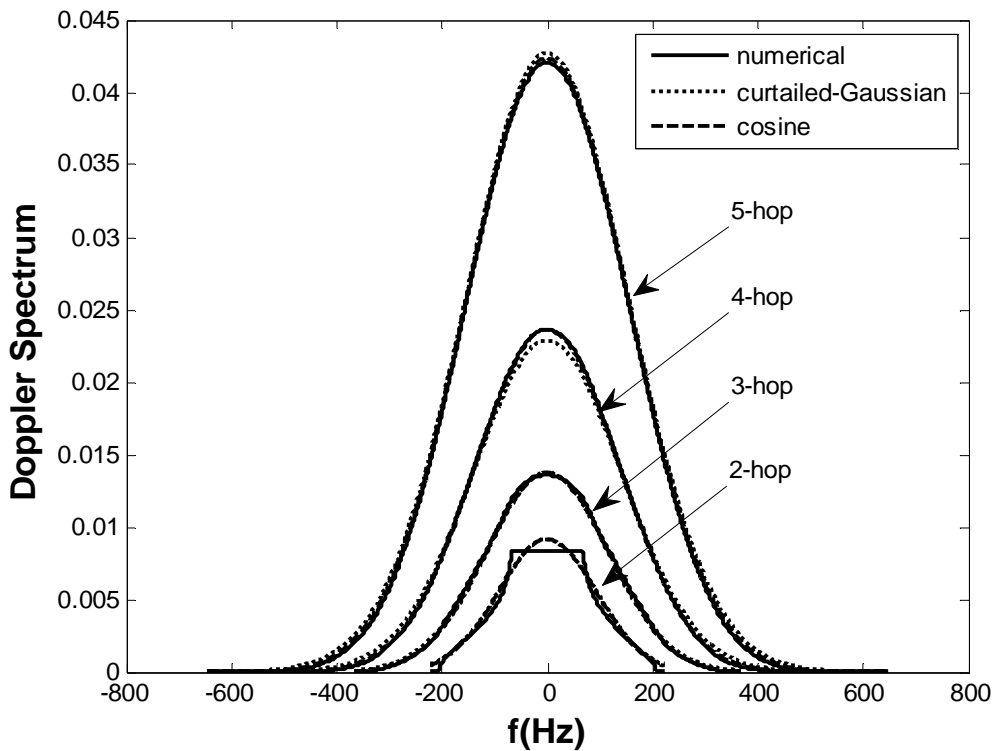


Figure 3. Validation result: The comparison of approximation models to the numerical value of Doppler spectrum ($f_1=70$ Hz)

Table 2. Kullback Leiber Divergence for curtailed-Gaussian & Cosine Models

number of hop	curtailed-Gaussian model	cosine model
$n = 2$	0.0135	0.0123
$n = 3$	0.0035	0.0016
$n = 4$	0.0031	0.0008
$n = 5$	0.0010	0.0005

5. Conclusions

From the discussion, the Doppler spectrum models of Multihop communication with mobile amplify-and-forward relays can be approximated with two simple functions, i.e., curtailed-Gaussian and cosine functions. Both models are valid if all mobile terminals are moving with almost identical velocity. It is shown that the approximation models can be used to generate mobile Multihop channels for evaluation of Doppler effects and to determine the delay spread in the study of cooperative diversity systems. Our future work is evaluating ICI in cooperative-OFDM due to high Doppler frequency introduced by Multihop mobile channel and how to mitigate the ICI in such environment.

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