

MCGDM with AHP based on Adaptive Interval Value Fuzzy

Yeni Kustiyahningsih^{*1}, Fatmawati², Herry Suprajitno³

¹Information System, Faculty of Engineering, Trunojoyo University, Madura, Indonesia

¹Information System, Department of Mathematics, FST, Airlangga University, Indonesia

^{2,3}Department of Mathematics, Faculty of Science and Technology, Airlangga University, Indonesia

^{*}Corresponding author, e-mail: ykustiyahningsih@trunojoyo.ac.id¹, fatmawati@fst.unair.ac.id², herry-s@fst.unair.ac.id³

Abstract

The purpose research is to develop the decision model of Multi-Criteria Group Decision Making (MCGDM) into Interval Value Fuzzy Multi-Criteria Group Decision Making (IV-FMCGDM), while the specific purpose is to construct decision-making model of Adaptive Interval Value Fuzzy Analytic Hierarchy Process (AIV-FAHP) uses Triangular Fuzzy Number (TFN) and group decision aggregation functions using Interval Value Geometric Means Aggregation (IV-GMA). The novelty research is to study the concept of group decision making by improving the middle point on the Interval Value Triangular Fuzzy Number (IV-TFN). It provides more accurate modeling, and better rating performance, and more effective linguistic representation. This research produced a new decision-making model and algorithm based on AIV-FAHP used to measure the quality of e-learning.

Keywords: group decision making, adaptive interval fuzzy, AIV FAHP, IVGMA, e-learning

Copyright © 2018 Universitas Ahmad Dahlan. All rights reserved.

1. Introduction

Multiple Criteria Group Decision Making (MCGDM) is a decision-making method to determine the best alternative from a set of alternatives with using alternative preferences as election criteria [1]. Some MCGDM methods include Simple Additive Weighting (SAW), Weighted Product (WP), Elimination and Choice Express Reality (ELECTRE), Technique for Order Preferences of Similarity Ideal Solution (TOPSIS), and Analytic Hierarchy Process (AHP) [2]. AHP is one of the most widely used MCGDM approaches. AHP is a structured multi-criteria technique for organizing and analyzing complex decisions based on many criteria. The AHP approach is able to elaborate complex multi-criteria problems into a hierarchical structure resulting in a flexible and easy-to-understand model. The AHP considers the value of logical consistency in the assessment, this logical concession is used to test the perception of the assessor and determine the optimal weighting in multi-criteria decision making [3-5].

The MCGDM method approach with AHP is not suitable for handling data that contains uncertainty. The issue of uncertainty can be attributed to incomplete information and unclear information [6]. The problems of measurements e-learning are imprecise data so that decision-makers can not provide appropriate numerical values for evaluation of criteria. The criteria in e-learning are subjective and qualitative, it is very difficult for decision makers to express appropriate preferences using numerical values. The weaknesses in the MCGDM method can be solved by using Fuzzy Multiple Criteria Group Decision Making (FMCGDM). The Fuzzy method in this study is used to accommodate the vague nature of decision-making for qualitative criteria. Fuzzy has excellent performance, more flexible decision-making processes, and capable of handling data that contains uncertainty and inaccuracy. Integration methods AHP with other methods can determine strategies more efficiently and give some contribution in decision making more optimal [7-10]. In group preferences, FMCGDM results in a higher consensus than non-Fuzzy decision makers, but the level of trust given to linguistic preference forms will overlap. Overlap on FMCGDM can be overcome by using Fuzzy Interval value.

Some previous researchers about interval value Fuzzy are decision-making model based on Interval Value Triangular Fuzzy Number by using extension Fuzzy TOPSIS for selection [11], Fuzzy analytic hierarchy process with Fuzzy type-2 interval sets [12], hybrid

FANP and IV FTOPSIS for network access selection [2], combine DEMATEL and TOPSIS based on interval Fuzzy [13, 14]. The study states that the Fuzzy Interval value is the expansion of Fuzzy, with the value of the Fuzzy membership function in the interval form. Fuzzy value intervals provide more accurate modeling and better rating performance. Intervals Fuzzy value have a more effective representation, have high flexibility, memory and time used for more efficient computation. This study has not modified interval point TFN with a different middle point, not yet used for implementation of e-learning measurement with group decision model. This study aims to construct Adaptive Adaptive Interval Value Fuzzy Analytic Hierarchy Process (AIV-FAHP) with improvement point on adaptive interval Fuzzy and aggregation of opinion with interval Value Geometric Means Aggregation (IV-GMA) to determine the weight of e-learning indicator. The measurement indicators are determined by Learning Technology System Architecture (LTSA) and ISO 9126 [15, 16]. The novelty research is to study the concept of group decision making with interval value Fuzzy to improve the point of the interval. At the interval value Fuzzy, each judge can determine the point of interval flexible and freely then developed into the concept of adaptive. Adaptive interval Fuzzy allows the appraiser to create its own set according to the required rules [17]. It can optimize the number and position of the Fuzzy set so using this method can improve the accuracy of recommendations [18]. Based on the description above, this research uses adaptive interval value Fuzzy approach for measurement e-learning with group decision-making model.

2. Research Method

2.1. Interval-Valued Fuzzy (IVF)

The interval-valued Fuzzy set A is defined as follows [11]:

$$A = \left\{ \left(x, \left[\mu_A^L(x), \mu_A^U(x) \right] \right) \right\}, \quad x \in X, \quad (1)$$

$$\begin{aligned} \mu_A^L, \mu_A^U: X &\rightarrow [0, 1]; \\ \forall x \in X, \mu_A^L(x) &\leq \mu_A^U(x). \end{aligned} \quad (2)$$

For Example,

$$\bar{\mu}_A(x) = \left[\mu_A^L(x), \mu_A^U(x) \right], \quad x \in X, \quad (3)$$

The interval value Fuzzy set A can be expressed as follows:

$$A = \left\{ \left(x, \bar{\mu}_A(x) \right) \right\}, \quad \bar{\mu}_A: X \rightarrow [0, 1]; \quad x \in X. \quad (4)$$

Based on these definitions, the interval value Fuzzy set A is represented by upper and lower limits.

2.2. Membership Function

The membership function is a curve showing the mapping of data input points into their membership values (often called membership degrees) that have intervals between 0 and 1. One way to use is the triangular membership function approach (Triangular Fuzzy Number). The function used in this study is Interval Value Triangular Fuzzy Number. Interval Value Triangular Fuzzy Number (IV-TFN) is given in Figure 1.

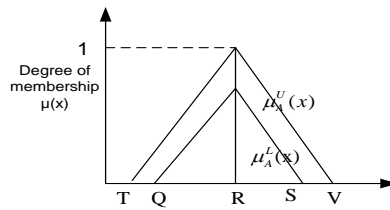


Figure 1. Interval value fuzzy set [19]

The membership function of IVF-TFN is defined as follows [19]:

$$\mu_A^L(x) = \begin{cases} (x-q)/(r-q); & q \leq x \leq r \\ (s-x)/(s-r); & r \leq x \leq s \\ 0; & \text{otherwise,} \end{cases} \tag{5}$$

With $A^L = (q, r, s), q \leq r \leq s$.

$$\mu_A^U(x) = \begin{cases} (x-t)/(r-t); & t \leq x \leq r \\ (v-x)/(v-r); & r \leq x \leq v \\ 0; & \text{otherwise,} \end{cases} \tag{6}$$

With $A^U = (t, r, v), t \leq r \leq v$.

2.3. Construct Interval Value Fuzzy Analytic Hierarchy Process (IV-FAHP)

This research uses Interval Value Triangular Fuzzy Number (IV-TFN) with improvement at the same middle point, different middle points, and aggregation of opinion with Interval Value Geometric Means Aggregation (IV-GMA). The steps in constructing IV-FAHP methods are as follows:

- a. Construction of pairwise comparison matrix D for criteria.

$$D = \begin{bmatrix} 1 & d_{12} & d_{13} & \dots & d_{1n} \\ d_{21} & 1 & d_{23} & \dots & d_{2n} \\ d_{31} & d_{32} & 1 & \dots & d_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ d_{n1} & d_{n2} & d_{n3} & \dots & 1 \end{bmatrix} \tag{7}$$

Where $i, j = 1, 2, \dots, n$.

- b. Normalization of pairwise comparison matrices. Each column is summed, then each element in the matrix is divided by the total value of the column. Next, determine the average row or vector that contains the set of numbers n weight w_1, w_2, \dots, w_n and consistency analysis.
- c. Representing the model of decision making in the Interval Fuzzy Triangular Number with the same middle point as in Figure 2.

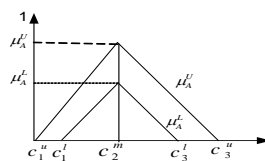


Figure 2. IV TFN with same middle point

Based on Figure 2. The matrix C can be expressed as follows:

$$C = \begin{bmatrix} 1 & c_{12} & c_{13} & K & c_{1n} \\ c_{21} & 1 & c_{23} & K & c_{2n} \\ c_{31} & c_{32} & 1 & K & c_{3n} \\ M & M & MO & M & \\ c_{n1} & c_{n2} & c_{n3} & K & 1 \end{bmatrix} \tag{8}$$

Where,

$$\mu_A^l \leq \mu_A^u, c_{2ij}^{ml} = c_{2ij}^{mu}, \quad c_{ij1}^l \leq c_{ij2}^m \leq c_{ij3}^l, \quad c_{ij1}^u \leq c_{ij2}^m \leq c_{ij3}^u$$

$$i, j = 1, 2, \dots, n.$$

d. Represents a model of decision making in a Fuzzy Triangular Number Interval with a different middle point at which point $g_2^{ml} < g_2^{mu}$ as shown in Figure 3.

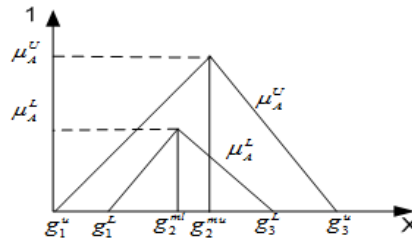


Figure 3. IV TFN different middle point

Based on Figure 3. The matrix G can be expressed as follows:

$$G = \begin{bmatrix} 1 & g_{12} & g_{13} & K & g_{1n} \\ g_{21} & 1 & g_{23} & K & g_{2n} \\ g_{31} & g_{32} & 1 & K & g_{3n} \\ M & M & MO & M & \\ g_{n1} & g_{n2} & g_{n3} & K & 1 \end{bmatrix} \tag{9}$$

Where,

$$g_{ij} = (g_{ij}^u, g_{ij}^l, g_{2ij}^{ml}, g_{2ij}^{mu}, g_{3ij}^l, g_{3ij}^u),$$

$$g_{ij}^{-1} = \left(\frac{1}{g_{3ij}^u}, \frac{1}{g_{3ij}^l}, \frac{1}{g_{2ij}^{ml}}, \frac{1}{g_{2ij}^{mu}}, \frac{1}{g_{ij}^l}, \frac{1}{g_{ij}^u} \right), \quad \mu_A^l \leq \mu_A^u, \quad g_{ij2}^{ml} < g_{ij2}^{mu}, \quad g_{ij1}^l \leq g_{ij2}^{ml} \leq g_{ij3}^l,$$

$$g_{ij1}^u \leq g_{ij2}^{mu} \leq g_{ij3}^u, \quad i, j = 1, 2, \dots, n.$$

Based on the pairwise comparison matrix that has been defined in Steps 3 and 4, then the matrix will be converted into Fuzzy interval number scale. The result of the respondent's assessment (group decision) on the pairwise comparison preference in the Fuzzy scale using the Interval Value Geometric Means Aggregation (IV-GMA). IV-GMA in the Interval Value Triangular Fuzzy Number (IV-TFN) with the same middle point is denoted by the Z matrix expressed as follows:

$$Z = \begin{bmatrix} z_{11} & z_{12} & z_{13} & \mathbf{K} & z_{1n} \\ z_{21} & z_{22} & z_{23} & \mathbf{K} & z_{2n} \\ z_{31} & z_{32} & z_{33} & \mathbf{K} & z_{3n} \\ \mathbf{M} & \mathbf{M} & \mathbf{MO} & \mathbf{M} & \\ z_{n1} & z_{n2} & z_{n3} & \mathbf{K} & z_{nm} \end{bmatrix} \tag{10}$$

Where

$$z_{ij} = \left(\left(\prod_{k=1}^n c_{ijk}^u \right)^{1/n}, \left(\prod_{k=1}^n c_{ijk}^l \right)^{1/n}, \left(\prod_{k=1}^n c_{ijk}^m \right)^{1/n}, \left(\prod_{k=1}^n c_{ijk}^l \right)^{1/n}, \left(\prod_{k=1}^n c_{ijk}^u \right)^{1/n} \right) \quad i, j = 1, 2, \dots, n.$$

e. Calculate the criterion weight of the matrix S with the same middle point. The weighted result of the S matrix criteria is denoted by U^* . The weight criteria for a Triangular Fuzzy Number (TFN) can be expressed as follows:

$$U^* = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \mathbf{M} \\ u_n \end{bmatrix}, \tag{11}$$

Where,

$$u_i = \left(\frac{\prod_{j=1}^n (z_{ij}^u)^{1/n}}{\sum_{j=1}^n z_{ij}^u}, \frac{\prod_{j=1}^n (z_{ij}^l)^{1/n}}{\sum_{j=1}^n z_{ij}^l}, \frac{\prod_{j=1}^n (z_{ij}^m)^{1/n}}{\sum_{j=1}^n z_{ij}^m}, \frac{\prod_{j=1}^n (z_{ij}^l)^{1/n}}{\sum_{j=1}^n z_{ij}^l}, \frac{\prod_{j=1}^n (z_{ij}^u)^{1/n}}{\sum_{j=1}^n z_{ij}^u} \right), \quad i, j = 1, 2, \dots, n.$$

f. Compute deFuzzyfication from u_i . DeFuzzyfication used to convert the Fuzzy output to a crisp value by the Best Non-Interval Fuzzy Performance (BNIP) method. BNIP can be stated as follows:

$$BNIP_i = \frac{a+b}{2},$$

$$a = \alpha_1 \left[\frac{(u_{3i}^u - u_{1i}^u) + (u_{2i}^{mu} - u_{1i}^u)}{3} + u_{1i}^u \right] \tag{12}$$

$$b = \alpha_2 \left[\frac{(u_{3i}^l - u_{1i}^l) + (u_{2i}^{ml} - u_{1i}^l)}{3} + u_{1i}^l \right]$$

Where $i = 1, 2, \dots, n$.

3. Results and Analysis

3.1. System Description

These steps are Modeling Stage, Modeling is the stage of identification of MCGDM problems by determining the number of variables to be used in the study (criteria, alternatives, appraisers, and respondents) as shown in Figure 4. The measurement indicators of e-learning in this study are determined based on Learning Technology System Architecture (LTSA) with adaptive design personalize and ISO 9126 [15-16].

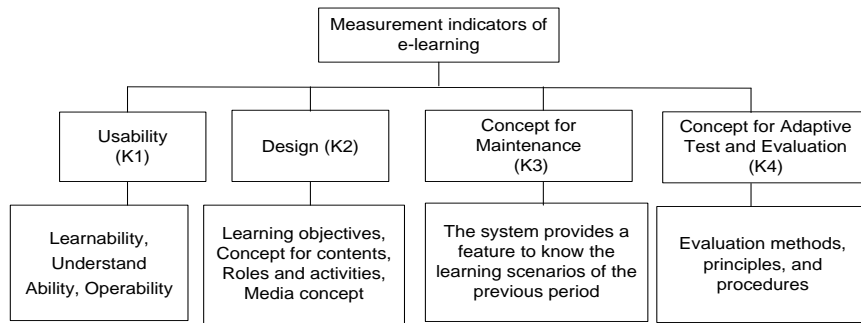


Figure 4. Indicator of e-learning

The next step, weighting using the concept of Adaptive Interval Value Triangular Fuzzy Number (AIV-TFN) with different interval points as shown in Figure 2. and 3. The last step is to construct AIV-FAHP methods such as at section 2.3. All research phases can be seen in Figure 5. AHP IVF Framework In this picture is described complete step Fuzzy adaptive interval framework.

3.2. Simulation and Analysis

After obtaining the mathematical model of the Interval Triangular Fuzzy Number with the same middle point and the different middle point, the simulation and analysis of the model have been made based on existing indicators in e-learning. This is done to determine the optimal solution in decision making based on the interval point and the smallest threshold value to determine the recommendation of e-learning system, e-learning mapping, and clustering/grouping e-learning.

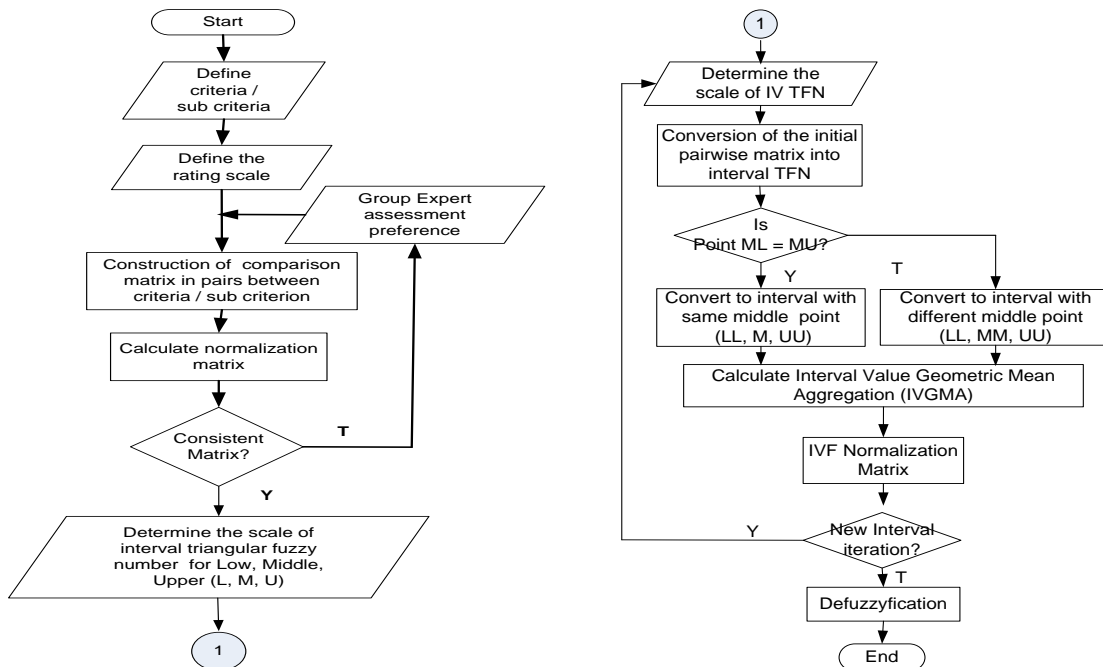


Figure 5. Framework AIV-FAHP

The stages of the simulation of this research program is:

- a. Doing questioners to some expert people to determine the comparative matrix assessment.

- b. Calculating the consistency matrix of pairwise matrices, if the consistency ratio (threshold) is less than 0.1 then the matrix is considered consistent.
- c. Determining the linguistic scale by using two different middle points on IVF TFN. The Next is determined the linguistic scale for each interval point. Table 1 shows the linguistic scale with the same middle point. Table 2 shows the linguistic scale with different midpoints. Each point is made dynamic, in accordance with existing rules.

Table 1. Linguistic Scale with the Same Middle Point

Numerical Scale	IV-TFN scale	Definition of Linguistic Variables
1	[(1,1), 1,(1,1)]	Equally Important
3	[(1, 1.5) 3 (3.5, 4)]	Slightly More Important
5	[(3, 3.5) 5 (5.5, 6)]	More important
7	[(5, 5.5) 7 (8, 8.5)]	Very Important
9	[(7, 7.5) 9 (9.5, 10)]	The most important

Table 2. Linguistic Scale with the Different Middle Point

Numerical Scale	IV-TFN scale	Definition of Linguistic Variables
1	[(1,1), (1,1), (1,1)]	Equally Important
3	[(1, 1.5) (3, 3.3) (3.5, 4)]	Slightly More Important
5	[(3, 3.5) (5, 5.3) (5.5, 6)]	More important
7	[(5, 5.5) (7,7.3) (8, 8.5)]	Very Important
9	[(7, 7.5) (9, 9,3) (9.5, 10)]	The most important

- d. Conversion of pairwise matrix matched into interval value, then normalize Weight in intervals with same middle point and Normalize Weight in intervals with different middle points.
- e. The next step is to determine the weight of the indicator assessment by performing DeFuzzyfication, in Table 3. It is DeFuzzyfication of the matrix with the same interval point, and Table 4. shows DeFuzzyfication of the matrix with different interval points.

Table 3. DeFuzzyfication with the Same Middle Point

Criteria	Lower limit	Upper limit	Defuzzification
K1	0,179	0,241	0,211
K2	0,288	0,383	0,336
K3	0,509	0,671	0,590
k4	0,957	1,247	1,102

Table 4. DeFuzzyfication with Different Middle Point

Criteria	Lower limit	Upper limit	Defuzzification
K1	0,097	0,079	0,088
K2	0,164	0,131	0,147
K3	0,313	0,243	0,278
k4	0,605	0,452	0,528

Based on the DeFuzzyfication results from each interval point at Table 3 and Table 4. It shows that with two-point interval yield a smaller value range between criterion one with another criterion. Therefore the authors conclude that the interval between two points has better accuracy than at one point. The result of weighting criteria also shows the same order ie Concept for Adaptive Test and Evaluation (K4), Concept for Maintenance (K3), Design (K2), Usability (K1). The result of comparison of criteria can be seen in Figure 6. Comparison of weight indicator. Fuzzy's adaptive interval concept can determine the optimal value by testing different interval points according to existing data. The methods discussed in this study can be applied in different domain problems, where the perceptions of decision-makers have an important role in the final outcome.

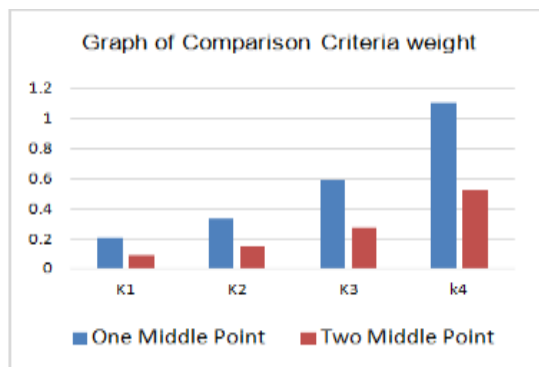


Figure 6. The Comparison of criteria weight

4. Conclusion

The interval value Fuzzy is an expansion of Fuzzy, with the value of the Fuzzy membership function in the interval form. Interval Value Fuzzy provide more accurate modeling, better rating performance, effective representation, and efficient computing. In the interval, the value Fuzzy is adaptive so that each judge can determine the point of interval flexible. FAHP is developed based on adaptive interval value by modifying the interval point on the Triangular Fuzzy Number with same middle point and different middle point. Decisions are taken in this study based on a smaller threshold, Improvements were also made at Interval Fuzzy points and opinion aggregation by Interval Value Geometric Means Aggregation (IV-GMA) method. Based on the simulation it is found that the range of values between the criteria generated with different TFN middle points is smaller than the same middle point. The methods in this study can be applied in different domain issues, where the perceptions of decision-makers have an important role in the final outcome.

The Further research can be developed using adaptive interval values FANP, ELECTRE, VIKOR, and DEMATEL with triangular Fuzzy or trapezoid Fuzzy number and aggregation of opinion with other methods.

References

- [1] Dursun M, Karsak E. A Fuzzy MCDM Approach for Personnel Selection. *Expert Systems with Applications*. 2010; 37(6): 4324-4330.
- [2] Skondras E, Sgora A. An ANP and Trapezoidal interval Value Fuzzy Technique for order preference by similarity to ideal solution Network access selection method. *Int. J. Commun. Syst.* 2014: 1-20.
- [3] Dagdeviren M, Yavuz S, Kilinc N. Weapon selection using the AHP and TOPSIS methods under Fuzzy environment. *Expert System with Applications*. 2009; 36: 8143-8151.
- [4] Aggarwal R, Sanjeet S. AHP and Extent Fuzzy AHP Approach for Prioritization of Performance Measurement Attributes. *International sciences index*. 2013; 7(1).
- [5] Kustiyahningsih Y, Rahmanita E, Purnama J. Integration Balanced Scorecard and Fuzzy Analytic Network Process (FANP) for Measuring Performance of Small Medium Enterprise (SME). *Journal of Theoretical and Applied Information Technology (JATIT)*. 2016; 94(2): 343-352.
- [6] Lazim A, Norsyahida Z. Integration of Fuzzy AHP and interval type-2 Fuzzy DEMATEL. *International Journal Expert Systems with Applications*. 2015; 42(9): 4397-4409.
- [7] Baohui J, Yuxin Z, Xiang L. Research on Zonal Inspection Intervals of Civil Aircraft Based on Improved FAHP. *Indonesian Journal of Electrical Engineering and Computer Science*. 2014; 12(1): 129-134.
- [8] Xie H, Duan W, Sun Y, Du Y. Dynamic DEMATEL Group Decision Approach Based on Intuitionistic Fuzzy Number. *TELKOMNIKA Telecommunication Computing Electronics and Control*. 2014; 12(4): 1064-1072.
- [9] Santoso I, Sa'adah M, Wijana S. QFD and Fuzzy AHP for Formulating Product Concept of Probiotic Beverages for Diabetic. *TELKOMNIKA Telecommunication Computing Electronics and Control*. 2017; 15(1): 391-398.
- [10] Diabagaté A, Azmani A, El Harzli M. Selection of the Best Proposal using FAHP: Case of Procurement of IT Master Plan's Realization. *International Journal of Electrical and Computer Engineering (IJECE)*. 2017; 7(1): 353-362.

-
- [11] Ashtiani B, Haghghirad F, Makui A, Ali Montazer G. Extension of FuzzyTOPSIS method based on interval-valued Fuzzy sets. *Applied Soft Computing*. 2009; 9(2): 457-461.
 - [12] Kahraman C, Ucal Sar I, Ostaysi B, Turanoglu E. Fuzzy Analytic Hierarchy Process with interval type-2 Fuzzy sets. *Knowledge-Based Systems*. 2014; 59: 48-57.
 - [13] Adil Baykasoglu A, Golcuk I. Development of an interval type-2 Fuzzy sets based hierarchical MADM model by combining DEMATEL and TOPSIS. *Expert Systems With Applications*. 2017; 70(15): 37-51.
 - [14] Kumar P, Claudio D. Implications of estimating confidence intervals on group Fuzzy decision making scores. *Expert Systems With Applications*. 2016; 65: 152-163.
 - [15] Cahyani A, Basuki, A, Malasari E, Kustiyahningsih Y. Design an Adaptive E-learning Application Architecture Based on IEEE LTSA Reference Model. *TELKOMNIKA Telecommunication Computing Electronics and Control*. 2015; 13(1): 284-289.
 - [16] Djouab R, Bari M, An ISO 9126 Based Quality Model for the e-Learning Systems. *International Journal of Information and Education Technology*. 2016; 6(5): 370-375.
 - [17] Sevarac Z, Devedzic V, Jovanovic J. Adaptive neuro-Fuzzy pedagogical recommender. *Expert Systems with Applications*. 2012; 39: 9797-9806.
 - [18] Ghavipour M, Meybodi M R. An adaptive Fuzzy recommender system based on learning automata. *Electronic Commerce Research and Applications*. 2016; 20: 105-115.
 - [19] Fuh CF, Jea R, Su JS. Fuzzy system reliability analysis based on level (k,1) interval-valued Fuzzy numbers. *Information Sciences*. 2014; 27(2): 185-197.