Low Complexity Multi-User MIMO Detection for Uplink SCMA System Using Expectation Propagation Algorithm

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Abstract

Sparse code multiple access (SCMA), which combines the advantages of low density signature (LDS) and code-division multiple access (CDMA), is regarded as one of the promising modulation technique candidate for the next generation of wireless systems. Conventionally, the message passing algorithm (MPA) is used for data detector at the receiver side. However, the MPA-SCMA cannot be implemented in the next generation wireless systems, because of its unacceptable complexity cost. Specifically, the complexity of MPA-SCMA grows exponentially with the number of antennas. Considering the use of high dimensional systems in the next generation of wireless systems, such as massive multi-user MIMO systems, the conventional MPA-SCMA is prohibitive. In this paper, we propose a low complexity detector algorithm named the expectation propagation algorithm (EPA) for SCMA. Mainly, the EPA-SCMA solves the complexity problem of MPA-SCMA and enables the implementation of SCMA in massive MU-MIMO systems. For instance, the EPA-SCMA also enables the implemantation of SCMA in the next generation wireless systems. We further show that the EPA can achieve the optimal detection performance as the numbers of transmit and receive antennas grow. We also demonstrate that a rotation design in SCMA codebook is unnecessary, which is quite rather different from the general assumption.

Keyword: SCMA, expectation propagation, low complexity, detection, MU-MIMO

1. Introduction

Next generation wireless networks are expected to satisfy tighter requirements such as massive connectivity, better quality of service, higher throughput, lower latency, and lower control signaling overhead, than the fourth generation system. These requirements can be met with new waveform and access designs [1]. As one of the most promising non-orthogonal multiple access candidate, sparse code multiple access (SCMA) has been addressed to cover these requirements. SCMA features the advantages of code division multiplexing (CDMA) and low density signature (LDS) [2].

SCMA is a modulation technique that directly modulates each group of binary data into a complex multidimensional codeword. This codeword is taken from a codebook [1]. At the receiver side, the message passing algorithm (MPA) can be implemented to achieve near optimal detection performance [3]. MPA calculates marginal distribution for each transmitted signal, conditional on received signal. The completeness probability information in each MPA's node results an outstanding performance of MPA. The sparsity of SCMA codeword makes a possibility to implement MPA on SCMA. However, the structure of MPA requires a recursive feedback message computation on every iteration. Therefore, the complexity of MPA detection grows exponentially with the codebook size.

To solve the complexity issue, many works have been completed either to reduce the complexity through the codebook design, using compressed sensing strategy [4, 5], or consider several extensions of the MPA, such as max-log MPA [6], SIC MPA [7], and even combined extensions of the MPA technique. However, these MPA-based detectors are suffering from an ex-
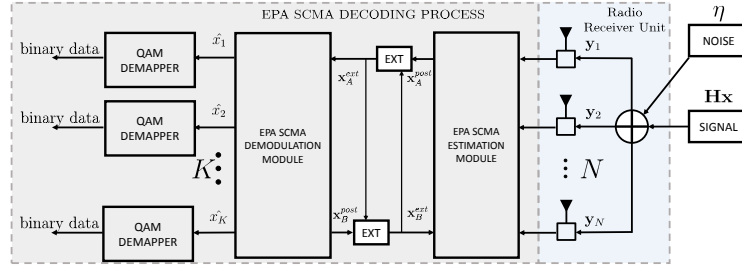
Figure 1. Block diagram of uplink scheme SCMA receiver

ponentially incremental complexity because the structure of MPA is still remained. Specifically, if the codebook size or the degree of freedom significantly increased, the MPAs for SCMA quickly becomes prohibitive due to its computational complexity.

In this work, we solve the complexity problem of the MPA-SCMA. We adopt the EPA in [8] and apply the EPA to the SCMA detection. The EPA approximates the marginal distribution of the posterior probability by using an exponential family. Therefore, the complexity of EPA is much lower than MPA.

With the theoretical verification, 1) we evaluate the EPA-SCMA performance and show that the EPA for SCMA can achieve near optimal detection performance . 2) We prove that appending a rotation value [9, 10] in SCMA encoder is unnecessary. The removal of the rotation value can omit many unnecessary calculations not only in decoding but also in encoding. Our hypotheses are also verified by the experimental results.

2. Research Method

In this section, we explain our system model and the implementation of EPA in SCMA for massive MU-MIMO systems detection scheme. Our system model is configured based on the original SCMA codebook as in [1]. The implementation of EPA to SCMA will be described afterwards. Finally, the complexity comparison of our EPA-SCMA and the original MPA-SCMA will be discussed.

2.1. System Model

We consider a SCMA system with U users operating on S orthogonal subcarriers. Each user equipment features N_t transmit antennas and the base station (BS) possesses N_r receive antennas. Let $K = UN_t$ and $N = SN_r$.

In the SCMA, each transmit symbol x_k is transmitted over S subcarriers using d degree, and different phase rotation values are introduced at different subcarriers [1]. For example, if $S = 4$ and $d = 2$, the mapping can be

$$\phi_k = [\phi_{k,s}] = \begin{bmatrix} e^{-j2\pi\Delta_1} \\ e^{-j2\pi\Delta_2} \\ 0 \\ 0 \end{bmatrix}, \mathbf{h}_k = \begin{bmatrix} \phi_{k,1}\mathbf{h}_{k,1} \\ \phi_{k,2}\mathbf{h}_{k,2} \\ \vdots \\ \phi_{k,S}\mathbf{h}_{k,S} \end{bmatrix}.$$

where $\Delta_i \in [0,1)$, $\mathbf{h}_{k,s} \in \mathbb{C}^{N_r}$ denote the channel vector from the k-th transmit antenna to the BS at the s-th subcarrier. Therefore, at the BS, the N-dimensional channel output vector $\mathbf{y}$ is expressed as $\mathbf{y} = \sum_{k=1}^{K} \mathbf{h}_k x_k + \boldsymbol{\eta}$, where $\boldsymbol{\eta}$ is the additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\sigma^2 \mathbf{I}$. $\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_K \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_K \end{bmatrix}^T$. Finally, we obtain

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\eta}. \tag{1}$$

The input-output relationship of the SCMA can be viewed as a MIMO communication system with K inputs and N outputs.

2.2. EPA for SCMA

Alg. 1: EPA-SCMA Algorithm

Initialization: $\gamma_{B\to A}^0 = 0, \lambda_{B\to A}^0 = \frac{1}{E_s}\mathbf{I}, d(\mathbf{Q}) = \mathrm{diag}(\mathbf{Q})$;

for $t = 1 : T_{max}$ **do**

  Estimation Module:

  (1) Compute the a posteriori mean/variance of $\mathbf{x}_A$:

$$\mathbf{v}_{A,t}^{\mathrm{post}} = \mathbf{\Sigma}^t = \left(\sigma^{-2}\mathbf{H}^H\mathbf{H} + d(\lambda_{B\to A}^{t-1})\right)^{-1} \tag{2a}$$

$$\mathbf{x}_{A,t}^{\mathrm{post}} = \mathbf{\Sigma}^t \left(\sigma^{-2}\mathbf{H}^H\mathbf{y} + \gamma_{B\to A}^{t-1}\right) \tag{2b}$$

  (2) Compute the extrinsic mean/variance of $\mathbf{x}_A$:

$$\mathbf{v}_{A,t}^{\mathrm{ext}} = \left(\frac{1}{d(\mathbf{\Sigma}^t)} - d(\lambda_{B\to A}^{t-1})\right)^{-1} \tag{3a}$$

$$\mathbf{x}_{A,t}^{\mathrm{ext}} = d(\mathbf{v}_{A\to B}^t)\left(\frac{\boldsymbol{\mu}^t}{d(\mathbf{\Sigma}^t)} - \gamma_{B\to A}^{t-1}\right) \tag{3b}$$

  Demodulation Module:

  (3) Compute the a posteriori mean/variance of $\mathbf{x}_B$:

$$\mathbf{x}_{B,t}^{\mathrm{post}} \leftarrow \mathsf{E}\{\mathbf{x}|\mathbf{x}_{A,t}^{\mathrm{ext}}, \mathbf{v}_{A,t}^{\mathrm{ext}}\} \tag{4a}$$

$$\mathbf{v}_{B,t}^{\mathrm{post}} \leftarrow \mathsf{Var}\{\mathbf{x}|\mathbf{x}_{A,t}^{\mathrm{ext}}, \mathbf{v}_{A,t}^{\mathrm{ext}}\} \tag{4b}$$

  (4) Compute the extrinsic mean/variance of $\mathbf{x}_B$:

$$\mathbf{v}_{B,t}^{\mathrm{ext}} = \lambda_{B\to A}^t = \left(\frac{1}{\mathbf{v}_{B,t}^{\mathrm{post}}} - \frac{1}{\mathbf{v}_{A,t}^{\mathrm{ext}}}\right)^{-1} \tag{5a}$$

$$\mathbf{x}_{B,t}^{\mathrm{ext}} = \gamma_{B\to A}^t = \left(\frac{\mathbf{x}_{B,t}^{\mathrm{post}}}{\mathbf{x}_{B,t}^{\mathrm{post}}} - \frac{\mathbf{x}_{A,t}^{\mathrm{ext}}}{\mathbf{v}_{A,t}^{\mathrm{ext}}}\right)^{-1} \tag{5b}$$

end

The input vector $\mathbf{x}$ to the equivalent MIMO channel $\mathbf{H}$ is a combined constellation $\Omega_1 \times \cdots \times \Omega_K$, where Ω_k is the set of constellation of the th transmission. Our target is to detect transmitted signals $\mathbf{x}$ over the received signals $\mathbf{y}$ in a given full knowledge of channel matrix $\mathbf{H}$. The complexity of the optimal detection grows exponentially with the size of the transmission and thus becomes prohibitive.

To solve this problem, we adopt the EPA in Alg.1 which is an iterative algorithm. As derived in our system model, the input-output relationship of SCMA can be configured as a MIMO communication system. Therefore, we can directly apply the EPA algorithm to the SCMA detector side. The detail of the EPA algorithm is the same as [8] and not mentioned in this paper due to limited space.

As shown in Figure 1b, the EPA-SCMA can be divided into two modules, i.e., estimation and demodulation. In the estimation module, given the channel model (1), we estimate $\mathbf{x}$ that minimizes the mean squared error (MMSE) given prior knowledge of $\mathbf{x}$. Let $(\mathbf{x}_A^{\mathrm{ext}}, \mathbf{v}_A^{\mathrm{ext}})$ and $(\mathbf{x}_B^{\mathrm{post}}, \mathbf{v}_B^{\mathrm{post}})$ be the input-output of demodulation module. The expectation and variance of the posterior estimator are computed as given by

$$\mathbf{x}_B^{\mathrm{post}} \leftarrow \mathsf{E}\{\mathbf{x}|\mathbf{x}_A^{\mathrm{ext}}, \mathbf{v}_A^{\mathrm{ext}}\}, \mathbf{v}_B^{\mathrm{post}} \leftarrow \mathsf{Var}\{\mathbf{x}|\mathbf{x}_A^{\mathrm{ext}}, \mathbf{v}_A^{\mathrm{ext}}\}. \tag{6}$$

Specifically, considering that $|\Omega_k| = M$, the expectations in (6) are with respect to $P(x_k|x_{A,k})$, which can be obtained by the Bayes rule

$$P(x_k|x_{A,k}) = \frac{P(x_{A,k}|x_m)P_x(x_m)}{P(x_{A,k})}, \tag{7}$$

where

$$P(x_{A,k}|x_m)P_x(x_m) = \frac{1}{M}\frac{1}{\pi v_{A,k}}\exp\left(-\frac{|x_{A,k}-x_m|^2}{v_{A,k}}\right), \tag{8}$$

$$P(x_{A,k}) = \frac{1}{M}\frac{1}{\pi v_{A,k}}\sum_{m=1}^{M}\exp\left(-\frac{|x_{A,k}-x_m|^2}{v_{A,k}}\right). \tag{9}$$

We then employ the performance analysis framework in [11] to develop the state evolution (SE) of the EPA-SCMA which is derived from a large scale system. For a large scale system, the extrinsic value of each EPA-SCMA module can be approximated by their average values, respectively. As given in [8], the input-output transfer function of estimation module can be derived from linear mean square error estimator, that is

$$v_A = \left(K^{-1}\text{tr}(\sigma^{-2}\mathbf{H}^H\mathbf{H} + v_B^{-1}\mathbf{I})^{-1}\right)^{-1} - v_B^{-1}. \tag{10}$$

where, $\text{tr}\{\cdot\}$ denotes trace operation, v_A is average extrinsic value of estimation module, and v_B is average extrinsic value of demodulation module. Furthermore, v_A can also be regarded as the SNR of the equivalent scalar additive white gaussian noise (AWGN) channel i.e. $y = x + v_A\eta$. Consistent with our assumption, where v_B and v_A are the average extrinsic values of demodulation and estimation EPA-SCMA modules, equivalent AWGN channel can be considered as a k-th channel under K users which has an identical channel value for every k-th user. Similarly, we define v as the scalar version of (6). Therefore, v can be calculated by $v = \text{Var}\{x|x_A, v_A\} = \text{E}\{|x - \text{E}\{x|x_A, v_A\}|^2\}$, where the expectation is with respect to $P(x|x_A)$ given by (7). Referring to [8], v_B can be defined as

$$v_B = \left(v^{-1} - v_A\right)^{-1}. \tag{11}$$

The iteration of the EPA is identical to the SE in (10) and (11) whose fixed points have MSE consistent with the MMSE from [12]. Moreover, the iteration of estimation and demodulation module can be traced from (10) and (11) without iterating the entire algorithm.

Finally, we compare the computational complexity of the three different algorithms: MPA, threshold-MPA [13], and EPA. Threshold-MPA-SCMA is recognized as a one of successful recently work to reduce the computational complexity of the original MPA-SCMA.

Table 1. Computational complexity comparison.

Comparison Setting1	$M = 4, N = 128, K = 196, I_t$=10, d=2
MPA	$\mathcal{O}(7.63617 \text{ X } 10^{64})$
Threshold-MPA	$\mathcal{O}(5.71086 \text{ X } 10^{64})$
EPA	$\mathcal{O}(62914560)$
Comparison Setting2	$M = 4, N = 64, K = 96, I_t$=10, d=2
MPA (BL)	$\mathcal{O}(1.24128 \text{ X } 10^{35})$
Threshold-MPA	$\mathcal{O}(9.25765 \text{ X } 10^{34})$
EPA	$\mathcal{O}(7864320)$

Table 1 shows the complexity orders of two settings. Let I_t denotes the number of Iteration. The implementation of MPA is prohibitive. Although threshold-MPA can decrease approximately 25% of the complexity, its implementation remains prohibitive. The EPA for SCMA successfully handles these situations, and its complexity is less than 10^{-20}% of the MPA complexity.
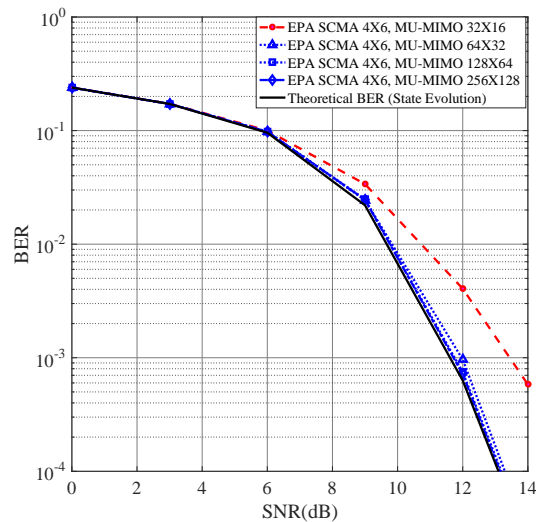
Figure 2. Performance analysis of EPA-SCMA

3. Result and Analysis

The simulation parameters are set as follows: the codebook size is $M = 4$ point codebook proposed in [2], the number of subcarrier is $S = 4$, and the number of multi-antennas users is $U = 6$. Each user has transmit antennas $N_t = 2$ and the BS has receiver antennas $N_r = 4$. As the SE is derived from a large scale system, we increase the transmit antennas and receiver antennas in the four following settings: 1) $N_t = 16$, $N_r = 32$, 2) $N_t = 32$, $N_r = 64$, 3)$N_t = 64$, $N_r = 128$, 4) $N_t = 128$, $N_r = 256$. In this way, we can observe the BER performance of EPA-SCMA from small to the large scale system. Channel used in this simulation is a normal random channel model. The rotation rule in codebook design is based on the codebook design proposed in [9].

Obviously, in small scale system as presented in Figure 3, MPA-SCMA which can be viewed as an optimal detector has a better performance than EPA-SCMA. However, as the numbers of transmit and receive antennas grow, EPA-SCMA performance improves significantly. Furthermore, EPA-SCMA successfully can match the theoretical BER performance as illustrated in Figure 2. As proved in [12], the theoretical performance can be viewed as near optimal performance. Therefore, the EPA-SCMA can achieve the near optimal performance. At the same time, under the parameter setting in Figure 2, we cannot evaluate the MPA-SCMA performance. We indicate that the complexity of MPA-SCMA increases extremely high, and becomes prohibitive to be implemented. For this reason, there is no MPA-SCMA BER performance can be presented in Figure 2 as MPA-SCMA fails to overcome its complexity problem.

Figure 3 also proves the argument on the need of putting a rotation value in the SCMA codebook as proposed in [1], [2], and [9]. Figure 3 describes that the BER performance between the EPA-SCMA and MPA-SCMA without rotation is identical to that with rotation. Consequently, the rotation value is unnecessary for the uplink scheme SCMA system. To support this argument, let the channel response on different users are vary and $\Delta_i = 0$ indicating that no rotation is included. Channel vector $\mathbf{h}_{k,s}$ for all k and s remains distinct. Therefore, no data interference occurs.

4. Conclusion

In this paper, we propose the EPA as the SCMA data detection which is solved the complexity problem of original MPA-SCMA. We proof that our EPA-SCMA can match the theoretical analysis, thus our EPA-SCMA achieve the near optimal performance. We also show that the rotation design in codebook is unnecessary in the uplink SCMA.
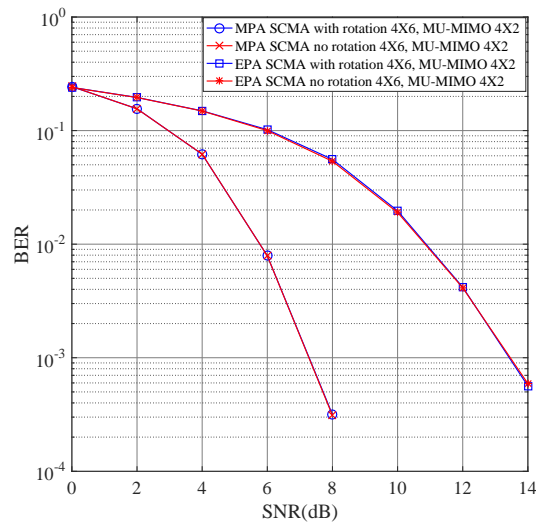
Figure 3. SCMA rotation and no rotation performance comparison

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