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# On Passband and Stopband Cascaded-Integrator-Comb Improvements Using a Second Order IIR Filter

**Alfonso Fernandez-Vazquez[1], Gordana Jovanovic Dolecek*[2]**
[1]Department of Electronics, School of Computer Engineering; ESCOM
National Polytechnic Institute; IPN, Mexico City, 07738, Mexico
[2]National Institute of Astrophysics, Optics and Electronics, Puebla, 72840, Mexico
e-mail: afernan@ieee.org,gordana@ieee.org

***Abstrak***

*Makalah ini mengusulkan sebuah tapis IIR orde-dua yang efisien untuk meningkatkan pita-lolos dan pita-henti dari tapis cascaded-integrator-comb (CIC). Dengan menggunakan dekomposisi fass-jamak pda tapis yang diusulkan, semua pentapisan dapat dipindahkan ke tingkat yang lebih rendah, dengan waktu D lebih rendah daripada laju input tinggi, di mana D adalah faktor penipisan. Tanggapan fasa keseluruhan dari CIC terkompensasi adalah mendekati linier pada pita-lolos. Parameter desain adalah nilai N tapis CIC tergandeng, faktor penipisan D, frekuensi pita-lolos $\omega$p, dan parameter terbobot $\alpha$.*

***Kata kunci:*** *CIC filter, filter CIC kompensasi, filter IIR, penipisan*

***Abstract***

*This paper proposes an efficient second order IIR filter which considerably improves the passband as well as the stopband of the cascaded-integrator-comb (CIC) filter. Using the polyphase decomposition of the proposed filter, all filtering can be moved to a lower rate, which is D times less than the high input rate, where D is the decimation factor. The overall phase response of the compensated CIC is approximately linear in the passband. The design parameters are the number of cascaded CIC filter N, the decimator factor D, the passband frequency $\omega_p$, and a weighted parameter $\alpha$.*

***Keywords***: *CIC filter, compensated CIC filter, IIR filter, decimation*

## 1. Introduction

The CIC (Cascaded Integrator Comb) filters are an economical class of linear phase FIR (Finite Impulse Response) filters [1]. They are used for decimation because of their simple multiplierless structure, i.e., this filter consists of cascaded integrators and combs, separated by a down-sampler (see Figure 1). The transfer function of the CIC filter is defined by

$$H(z) = \left(\frac{1}{D}\frac{1-z^{-D}}{1-z^{-1}}\right)^{N}, \tag{1}$$

where *D* and *N* are the decimation factor and the number of stages, respectively. Figure 2 illustrates the resulting magnitude characteristic of the CIC filter, where $\omega_p$ is the passband edge frequency and the folding bands are the bands around the zeros of the CIC filter.

Observe that the CIC filter has a poor magnitude response, that is, a high passband droop and a low attenuation in the folding bands. There have been proposed different methods for compensating the passband droop as well as for improving the stopband characteristics, for example [2-8]. Sharpening-based methods [5] generally improve both the passband and the stopband characteristic of the CIC filter as an expense of the increased complexity. The generalized CIC filter (GCF), which considerably improves the folding band attenuation has been proposed in [6]. Methods for the passband improvement of the GCF are given in [7].

The main goal of this paper is to introduce an efficient second order IIR filter to improve the passband as well as the stopband of a CIC filter. To this end, one pair of the complex-conjugated zeros, and one pair of the complex-conjugated poles, are placed into the first folding band near the passband edge, respectively.

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The rest of the paper is organized as follows. The proposed IIR compensator filter is described in Section 2. Section II also describes the magnitude response optimization. Section 3 introduces an efficient structure of the overall decimation filter. Discussion of the results is given in section 4.

## 2. Proposed IIR Filter

The proposed second order IIR filter is given by

$$P(z) = A\frac{(1-c_1z^{-1})(1-c_2z^{-1})}{(1-d_1z^{-1})(1-d_2z^{-1})} \tag{2}$$

where $c_1$ and $c_2$ are the pair of the complex conjugated zeros and $d_1$ and $d_2$ are complex conjugated poles, and $A$ is the normalization constant. We define $c_1$, $c_2$, $d_1$, and $d_2$ as follows

$$c_{1,2} = e^{\pm j\omega_0},\ d_{1,2} = re^{\pm j\omega_1}.) \tag{3}$$

The frequency $\omega_0$ is placed inside the first folding band, while the frequency $\omega_1$ is located near the passband edge frequency. Using (2) the relation of (3) becomes

$$P(z) = A\frac{1-2cos\omega_0z^{-1}+z^{-2}}{1-2rcos\omega_0z^{-1}+r^2z^{-2}}. \tag{4}$$

The resulting overall transfer function is expressed as

$$G(z) = H(z)P(z). \tag{5}$$

Moreover, the frequency response is given by

$$G(e^{j\omega}) = \left(\frac{1}{D}\frac{\sin(D\omega/2)}{\sin(\omega/2)}\right)^N \frac{2A(cos\omega-cos\omega_0)e^{j(1-D)N\omega/2}}{(1+r^2)cos\omega_0-2rcos\omega_1+j(1-r^2)sin\omega_0}. \tag{6}$$

Generally, the phase response of (6) is not linear because the filter $P(z)$ is an IIR filter. However, for small values of $\omega_p$, it is straightforward to show that the phase response is approximately linear in the band [0, $\omega_p$] and it is given by

$$\phi(\omega) \approx \frac{(1-D)N\omega}{2}, \tag{7}$$

Next issue is to find the optimum values $\omega_0$, $\omega_1$, and $r$. This is described in next subsection.

### Optimal Values forf $\omega_0$, $\omega_1$, and $r$

We define the error function in the passband region as

$$E_p(\omega) = \left|G(e^{j\omega})\right| - 1, 0 \le \omega \le \omega_p. \tag{8}$$

Similarly, the error function in the stopband region is given by

$$E_s(\omega) = \left|G(e^{j\omega})\right|,\ \omega_s \le \omega \le \pi. \tag{9}$$

The objective function defined as a convex combination of (8) and (9) is

$$\epsilon = \alpha\int_0^{\omega_p} E_p(\omega)d\omega + (1-\alpha)\int_{\omega_s}^{\pi} E_s(\omega)d\omega \tag{10}$$

where $\alpha$ is a weight parameter and $\omega_s$ is equal to $3\pi/2D$.

Therefore, the optimization problem is formulated as

$$\min_{\omega_0,\omega_1,r} \varepsilon \quad \text{subject to } r<1 \tag{11}$$

The constraint ensures the stability of the IIR filter, i.e., the poles must be inside the unit circle. We use the MATLAB function *fmincon* to solve (11).

Following example illustrates the design of the proposed improved CIC filter.

**Example 1**. Consider the decimation factor $D$ equals 32, the number of stages $N$ is 5, the passband edge frequency is $0.65\pi/D$ and $\alpha = 0.8$.

The resulting values for $\omega_0$, $\omega_1$, and $r$ are, respectively, 0.245437, 0.06695159, and 0.986255. Consequently, the system function $P(z)$ in this example is as follows:

$$P(z) = 0.076883 \frac{1-1.940062z^{-1}+z^{-2}}{1-1.96809z^{-1}+0.972698z^{-2}} \tag{12}$$

The magnitude response of the filter P(z) along with the CIC filter response are shown in Figure 3(a). Figures 3(b) and 3(c) show the passband zoom and the phase response in the passband, respectively.

As expected, the proposed method improves the passband region (see dashed line in Figure 3(b)) and stopband region as well (see dashed line in Figure 3(a)) of the CIC filter. Observe that the phase response is approximately linear in the passband.

In next section, we present an efficient implementation structure of the proposed filter.

## 3. Efficient Decimation Filter Structure

This section focuses on the efficient structure of the IIR filter $P(z)$ based on the method presented in [9].

Consider the following identity [9]:

$$\frac{1}{1-dz^{-1}} = \frac{\sum_{n=0}^{D-1} d^n z^{-n}}{1-d^D z^{-D}} \tag{13}$$

Combining (2) and (13), the system function $P(z)$ becomes

$$P(z) = A\frac{N(z)}{(1-d_1^D z^{-D})(1-d_2^D z^{-D})} \tag{14}$$

where

$$N(z) = (1-c_1 z^{-1})(1-c_2 z^{-1}) \sum_{n=0}^{D-1} d_1^n\, z^{-n} \sum_{n=0}^{D-1} d_2^n\, z^{-n} \tag{15}$$

Expressing $N(z)$ in a polyphase form [10], we have:

$$N(z) = \sum_{k=0}^{D-1} z^{-k} N_k(z^D) \tag{16}$$

Consequently, the IIR filter system function $P(z)$ is get as shown below

$$P(z) = A\frac{\sum_{k=0}^{D-1} z^{-k} N_k(z^D)}{(1-d_1^D z^{-D})(1-d_2^D z^{-D})} \tag{17}$$

Figure 4(a) shows the resulting decimation structure based on (17). Applying multirate identities [9] to (17), one can move the polyphase components $N_k(z)$ of the denominator of $P(z)$ to a lower sampling rate as shown in Fig. 4(b).

According to [9], the number of multiplies needed to calculate one output sample of the structure in Figure 4(a) (multiplies per output sample, MPOS) from the input of $P(z)$ is $5D$, while using the structure of Figure 4(b) the number of MPOS is $3 + 2D$.

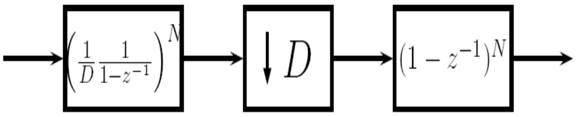



Figure 1. Cascaded-integrator-comb (CIC) filter

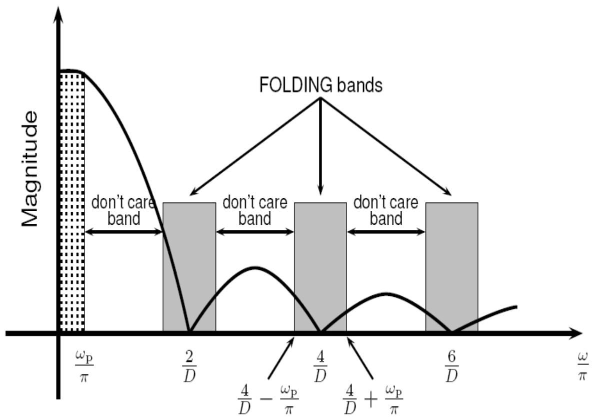
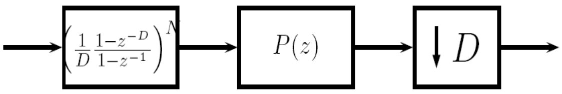
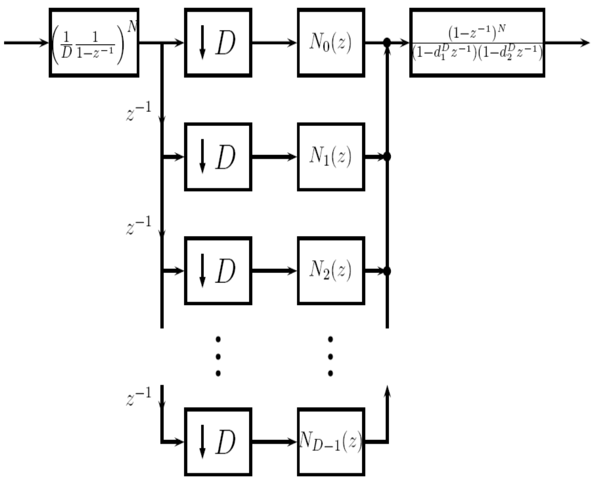



Figure 2. Magnitude Characteristic of the CIC Filter

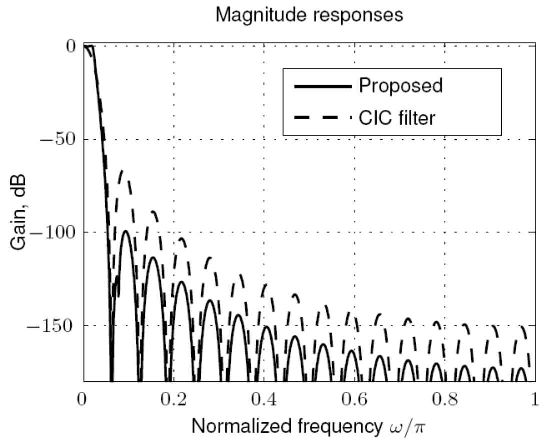



(a)

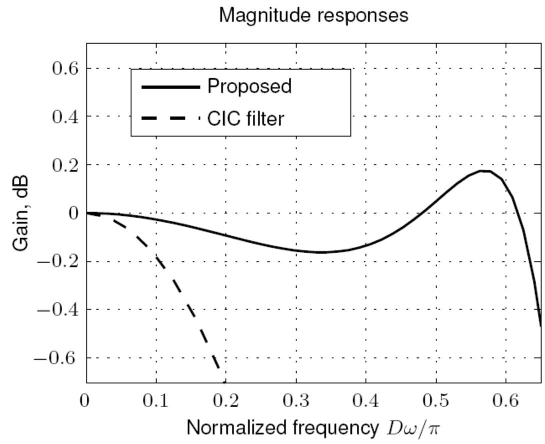



(b)

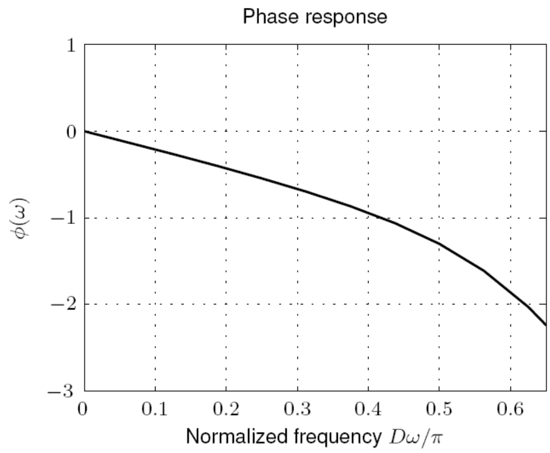
Figure 4. Efficient decimation structure




(a)




(b)




(c)

Figure 3. Frequency response of the designed IIR compensation filter in Example 1

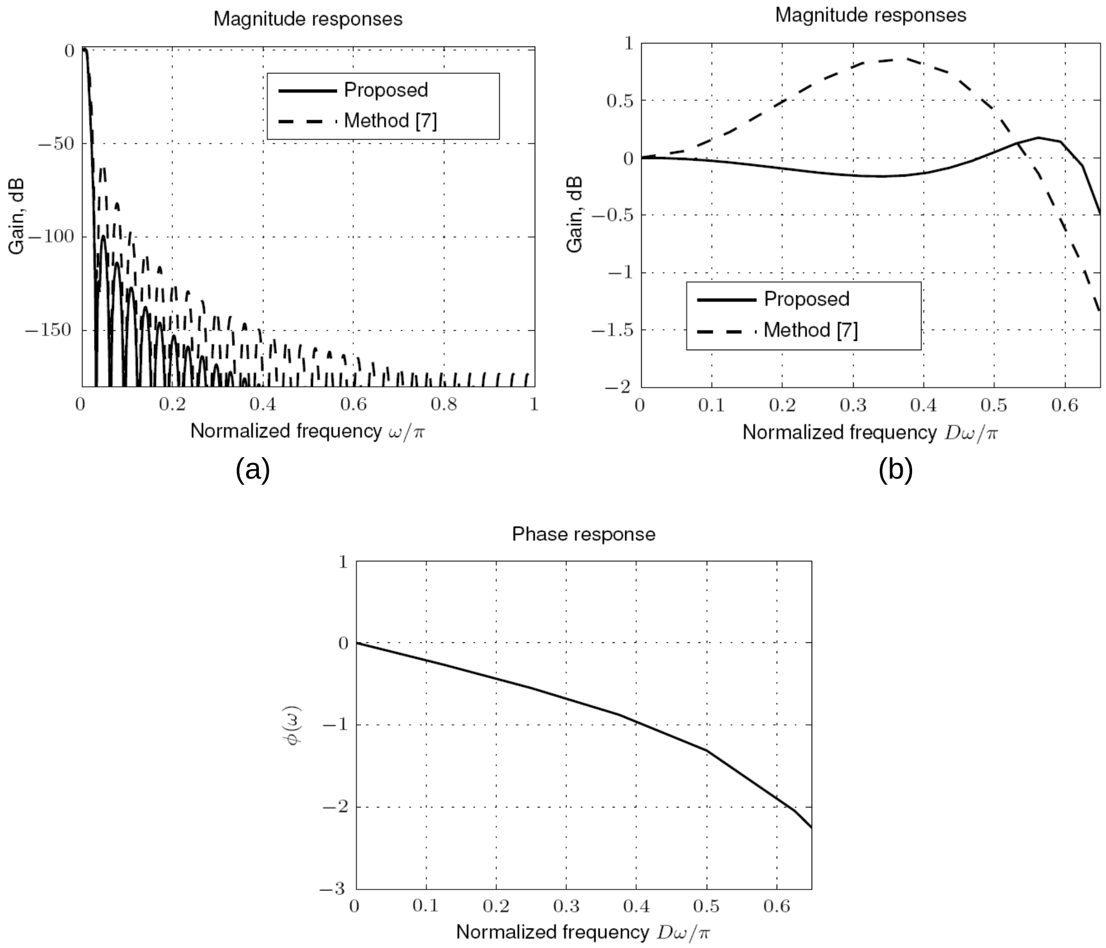
## 4. Discussion

In order to evaluate the performance of the proposed decimator filter, we compare our approach with the method recently proposed in [7]. The method [7] uses the generalized CIC filter (GCF) to improve the stopband and a second order FIR filter to improve the passband region. Next example compares the proposed method and the method [7].

**Example 2**. In this example, the design parameter are $D$ = 64, $K$ = 5, $\omega_p$= 0.65$\pi/D$ and $\alpha$ = 0.8. Applying minimization to (10), the optimum values of $\omega_0$, $\omega_1$, and $r$ are 0.122718, 0.033469, and 0.993108, respectively. The corresponding system function $P(z)$ is given by

$$P(z) = 0.077117\ \frac{1-1.984959z^{-1}+z^{-2}}{1-1.985104z^{-1}+0.966692z^{-2}}. \tag{18}$$

Figures 5(a) and 5(b) shows the corresponding magnitude responses. Note that the proposed method exhibits better magnitude characteristic in both passband and stopband regions than method [7]. Figure 5(c) shows the phase response in the passband thus confirming that the phase is approximately linear in the passband.




(c)

Figure 5. Comparison with the method [7]

## 5. Conclusion

This paper presents a second order IIR filter which considerably decreases the passband droop and increases the attenuation in the folding bands of the CIC filter. Using the multirate identity the filter can be moved to a lower rate which is $D$ times less than the high input rate. The design parameters are the number of cascaded CIC filter $N$, the decimator factor $D$, the passband frequency $\omega_p$, and the weighted parameter $\alpha$. The comparison with the recently proposed method [7] shows that the proposed method has a better magnitude response. Additionally it provides less complexity. It requires 3 multipliers comparing with the method [7] which requires 2 + $\lfloor D/2 \rfloor$ multipliers; 2 for compensator and $\lfloor D/2 \rfloor$ for GCF filter. The stability problem is solved with the constraint in the optimization procedure that the pole must be inside the unit circle. Even though that an IIR filter generally has a nonlinear phase the proposed filter has an approximately linear phase in the band defined with the passband frequency.

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