

Initial Phase Effect on DOA Estimation in MMIMO Using Separated Steering Matrix

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Abstract

Providing simple and low complexity algorithms for estimating the direction of arrival in large systems using Massive MIMO is considered an important issue. In this paper a method with reduced complexity was proposed to estimate the direction of arrival in FD-MMIMO. The Separated Steering Matrix (SSM) algorithm uses two separated equations for estimating elevation and azimuth angles of Multi-users. This method reduces the complexity of calculating the covariance matrix by decreasing the size of this matrix. This technique is tested using 2D-MUSIC algorithm. Since the mobility of devices affects the accuracy of direction estimation, thus the effect of the initial phase of transmitted signal from mobile device is tested.

Keywords: Massive multi-input multi-output, full dimension (FD), direction of arrival (DOA), initial phase

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1. Introduction

Multiple-Input Multiple-Output (MIMO) contains multiple number of transmitters and receivers antenna that increase data rate needed to provide services [1]. Massive multiple-input, multiple-output, or massive MIMO is an extension of MIMO, which presents a large number of antennas grouped together at the transmitter and receiver to enhance throughput and spectrum efficiency. Additional antennas provide huge improvement in throughput by focusing energy into ever smaller regions of space [2].

MMIMO systems differ from traditional MIMO systems not only in the number of antenna elements, but also these systems supposed to provide high data rates, reduced error probabilities, reduced noise, and lower transmit power per antenna. All above become even more impressive in the multi-user scenario where the base station transmits to several users simultaneously [3]. Estimating the direction in two dimensional (2D) MMIMO (the azimuth and elevation angles) represent a challenge that pinpoint the attention in the field of array processing [4]. Among DOAs methods, Multiple Signal Classification (MUSIC) is a well known technique based on eigenvalues decomposition of an array correlation matrix. MUSIC's advantages are high-resolution capability, simplicity and low computational load [2].

Such solutions when used with MMIMO involve rather high computational complexity due to the large size of covariance matrix. Hence, development of reduced-complexity DOA estimation algorithms for FD-MMIMO systems becomes necessary from a practical implementation perspective. Thus, a new method named 'Separated Steering Matrix (SSM)' algorithm is proposed. SSM decreases the covariance matrix size of the received signal, which represents the big reduction in complexity. Other researchers proposed methods to reduce the complexity of DOA estimation.

Xiaoyu Li in [2] proposed a propagator method (PM) DOA estimation algorithm based on 2D Massive MIMO. The PM estimation method tested using ESPRIT method. Porozantidou and Chryssomallis in [5] determine the correct azimuth and elevation angles of signal wave forms impinging on a special L-shape antenna array, consisting of two array branches placed on x and y axes. Xiao Yang, et. al in [6] produced new version of PM, by partitioning the covariance matrix of the received signal, such that the reconstruction of system model is not needed, so the complexity further reduced. Yasmine M., et.al. in [7] produced a comparative study in DOA algorithms used in MIMO system. MUSIC and ESPRIT produced best estimation time. Another important issue that needs to be investigated is the effect of the initial phase of the signal

transmitted from multi mobile devices. The work in this paper is focused on the effect initial phase on DOA estimation of received signals in FD-MMIMO.

2. Research Method

The model of MMIMO system is represented by 2D matrix with M element in the x-dimension and N element in the y-dimension. T different sources transmit signal S_i at the same time, where $1 \leq i \leq T$ as in Figure 1.

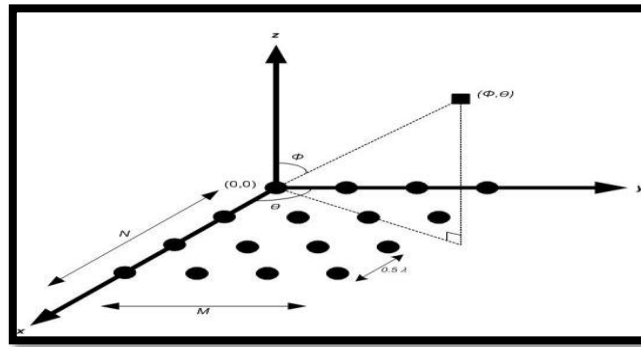


Figure 1. Array Geometry configuration for 2-D direction-of-arrival (DOA) Estimation [8]

The received signal at (m, n) th antenna, is represented by the following equation

$$X_{m,n} = \sum_{i=1}^T S(i) e^{j((m-1)u_i + (n-1)v_i) + \tau} + W_{m,n} \tag{1}$$

$1 \leq m \leq M, 1 \leq n \leq N$, and τ is initial phase in degrees, where

$$u_i = \left(\frac{2\pi d}{\lambda}\right) \sin \theta_i, v_i = \left(\frac{2\pi d}{\lambda}\right) \cos \theta_i \sin \phi_i \tag{2}$$

and $W_{m,n}$ is the effect of Rayleigh fading channel with AWGN (zero mean and variance σ^2) [9].

$$f_x(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} \tag{3}$$

after collecting the received signal in M x N matrix, the final received signal from all sources

$$X = \sum_{i=1}^T S_i a(u_i) a^T(v_i) + W \tag{4}$$

where $a(u_i) = [1, e^{ju_i}, \dots, e^{j(M-1)u_i}]^T$, and $a(v_i) = [1, e^{jv_i}, \dots, e^{j(N-1)v_i}]^T$ are the steering vectors for elevation and azimuth angles of all sources. W is M x N noise matrix. We can rewrite (1) equation into the following form

$$X = AS + W \tag{5}$$

where $S = [S_1, S_2, \dots, S_T]^T$ and $A = [a_1, a_2, \dots, a_T]$ with $a_i = a(u_i) \otimes a(v_i)$, and $1 \leq i \leq T$. \otimes denotes Kronecker product [10]

$$A = \begin{pmatrix} 1 & 1 & 1 \\ e^{j(U_1)} & e^{j(U_2)} & e^{j(U_T)} \\ \vdots & \vdots & \vdots \\ e^{j((M-1)U_1)} & e^{j((M-1)U_2)} & e^{j((M-1)U_T)} \\ e^{j(V_1)} & e^{j(V_2)} & e^{j((M-1)V_T)} \\ \vdots & \vdots & \vdots \\ e^{j((M-1)U_1+V_1)} & e^{j((M-1)U_2+V_2)} & e^{j((M-1)U_T+V_T)} \\ \vdots & \vdots & \vdots \\ e^{j((N-1)U_1)} & e^{j((N-1)U_2)} & e^{j((N-1)U_T)} \\ \vdots & \vdots & \vdots \\ e^{j((M-1)U_1+(N-1)V_1)} & e^{j((M-1)U_2+(N-1)V_2)} & e^{j((M-1)U_T+(N-1)V_T)} \end{pmatrix} \tag{6}$$

3. The Proposed Method

3.1. Separated Steering Method

The proposed new algorithm for estimating the direction of arrival of multi-user FD-MMIMO is based on separating the input signal into two signals. The first signal is used to estimate the elevation angle only then use it along with the second part of input signal in estimating the azimuth angle. This separation is based on Separated Steering Matrix (SSM). The procedure goes along the following steps:

First: find X_u and X_v from the received signal

$$X_u = A_u S + W \quad \text{and} \quad X_v = A_{uv} S + W$$

Where

$$A_u = \begin{pmatrix} 1 & 1 & \dots & 1 \\ e^{j(U_1)} & e^{j(U_2)} & \dots & e^{j(U_T)} \\ \vdots & \vdots & \dots & \vdots \\ e^{j((M-1)U_1)} & e^{j((M-1)U_2)} & \dots & e^{j((M-1)U_T)} \end{pmatrix} \tag{7}$$

And

$$A_{uv} = \begin{pmatrix} e^{j(V_1)} & e^{j(V_2)} & e^{j((M-1)V_T)} \\ \vdots & \vdots & \vdots \\ e^{j((M-1)U_1+V_1)} & e^{j((M-1)U_2+V_2)} & e^{j((M-1)U_T+V_T)} \\ \vdots & \vdots & \vdots \\ e^{j((N-1)U_1)} & e^{j((N-1)U_2)} & e^{j((N-1)U_T)} \\ \vdots & \vdots & \vdots \\ e^{j((M-1)U_1+(N-1)V_1)} & e^{j((M-1)U_2+(N-1)V_2)} & e^{j((M-1)U_T+(N-1)V_T)} \end{pmatrix} \tag{8}$$

As we can see the steering matrix can be separated into two matrices A_u and A_{uv} . The matrix A_u include only steering vector corresponding to elevation angle θ , While the A_{uv} contain both azimuth and elevation angles.

Second: compute the correlation matrix R corresponding to each part of the input signal, X_u, X_{uv}

$$R_{X_u} = E[X_u X_u^H] = A_u S A_u^H + \sigma^2 I \quad \text{and} \quad R_{X_{uv}} = E[X_{uv} X_{uv}^H] = A_{uv} S A_{uv}^H + \sigma^2 I \tag{9}$$

Third: perform the Eigen decomposition on both $R_{X_u}, R_{X_{uv}}$ correlation matrix to obtain the matrix V_u and V_{uv} whose columns are the eigenvectors corresponding to the smallest eigenvalues of $R_{X_u}, R_{X_{uv}}$.

Fourth: Search for the spectral peaks in the MUSIC [10] spectrum to find the elevation angle $\{\theta_1, \theta_2, \dots, \theta_T\}$

$$P(\theta_i) = \left\| \frac{1}{A_u^H V_u V_u^H A_u} \right\| \tag{10}$$

Fifth: after estimating the elevation angle, now search the spectral peaks in MUSIC spectrum to find azimuth angles $\{\varphi_1, \varphi_2, \dots, \varphi_T\}$ corresponding to the estimated elevation angles [10].

$$P(\theta_i, \varphi_i) = \left\| \frac{1}{A_{UV}^H V_{UV} V_{UV}^H A_{UV}} \right\| \tag{11}$$

3.2. Beam Width of Linear Array

The antenna electric field pattern of array antenna can be given by [11]

$$|E(\theta)| = \left| \frac{\sin[N(\pi d/\lambda) \sin \theta]}{\sin[(\pi d/\lambda) \sin \theta]} \right| \tag{12}$$

where N is the number of antenna elements, d is the spacing between antenna elements. In order to find the beam width (3 dB), the above equation should be equated to $\frac{1}{\sqrt{2}}$ and solve for θ . The solution will come to be as $0.89 \frac{\lambda}{D}$. Where D is the total aperture distance and can be approximated as ND . For spacing of $D=0.5\lambda$ the equation simplifies and can be approximated as $2/N$, Thus beam width of 2D antenna can be represented as [12]

$$\Delta\theta_{3dB} = 1.78/N \text{ (rad)} \quad \text{or} \quad \Delta\theta_{3dB} = 102/N \text{ (deg)} \tag{13}$$

4. Complexity Calculation and Comparison

The complexity of the proposed algorithm compared to methods from literature is shown in Table 1. Where M, N denote the number of elements in the dimension of X and Y , respectively; Snap denote the number of snapshots and T the number of signal sources. Figure 2 shows the number of operations required in each method. The figure views that SSM method uses less multiplication methods than traditional method, while in addition both methods produce approximately the same number of operations.

Table 1. Complexity Comparison (T is the number of source signals, M, N are the number of antennas elements, Snap is the No. of samples)

No. of Multiplications in traditional method	$T^2 \times (MN+1) + (MN)^2 \times (1+T) + (MN+1) \times ((MN)^2 + (2MN)) + (360 \times 90 \times \text{Snap}) \times (1+(MN-T)) \times (1+2MN) + MN+4(M+N)-10$
No. of Additions in traditional method	$(T-1) \times ((MN)^2 \times (Tx(MN)) + (MN-1)^2 \times (MN-1)) + (360 \times 90 \times \text{Snap}) + ((MN-1) \times (MN-T) \times 2 + (MN-T-1))$
No. of Multiplications in SSM method	$T^2 \times (MN+1) + (MN)^2 \times (1+T) + (MN+1) \times ((MN)^2 + (2MN)) + (M-1) \times (M^2+2M) + T \times (360 \times \text{Snap}) \times (1+(M-T) \times (1+2M)) + (90 \times \text{Snap}) \times (1+(MN-M-T)) \times (1+2(MN-M)) + MN+4(M+N)-10$
No. of Additions in SSM method	$(T-1) \times ((MN)^2 \times T \times (MN)) + (MN-1)^2 \times (MN+1) + ((M-1)^2 \times (M+1)) + T \times ((360 \times \text{Snap}) \times (1+(M-1) \times 2(M-T) + (M-T-1)) + (90 \times \text{Snap}) \times ((MN-M-1) \times (MN-M-T) \times 2 + (MN-M-T-1))$

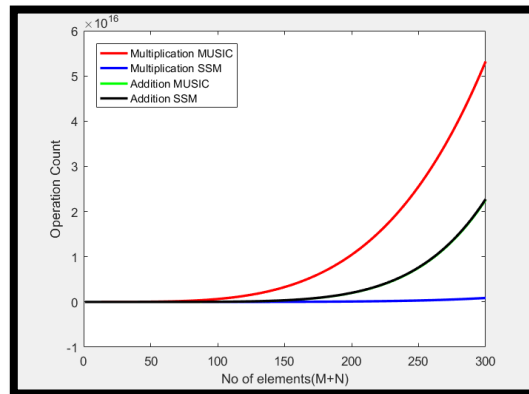


Figure 2. Operation Count comparison with different No of antenna elements ($T=6, \text{Snap}=100$)

Another comparison is made with methods proposed in [2] and [6]. Table 2 shows the complexity comparison. The comparison views that SSM and improved PM both surpasses the PM method and produces comparable results.

Algorithm	No. of elements	Complexity
PM [2]	20	10e6
	100	10e10
Improved PM [7]	100	10e6
	600	10e7
Proposed SSM	100	10e6
	600	10e7

5. Results and Discussion

To test the performance of the new proposed method, the algorithm is tested using MATLAB 2016a. In the simulation the number of elements selected to be $M=N=20$ with 0.5λ spacing between elements, number of snapshots=50, and carrier frequency=26 GHz. Figure 3 shows the array geometry. It also shows that the spacing is in term of mmWave.

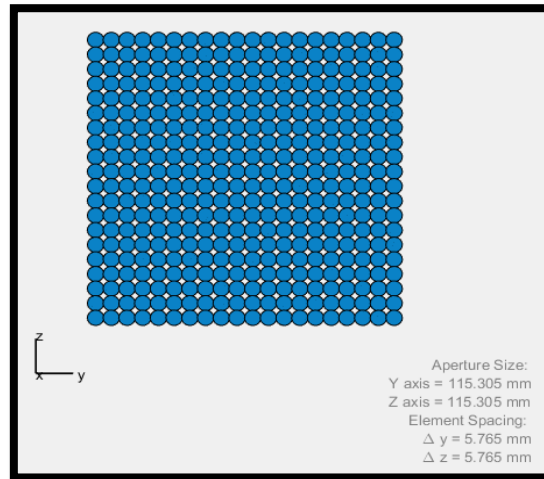


Figure 3. Massive MIMO Array Geometry

Five signals have been received from different mobile devices coming from different angles under the effect of Rayleigh fading channel as in equation (3) with zero mean and $\sigma^2=0.1$. The direction of the received signals is defined with azimuth angles $\varphi = \{-30^\circ, 70^\circ, -10^\circ, 30^\circ, 20^\circ\}$ and elevation angles $\theta = \{70^\circ, 40^\circ, -30^\circ, -10^\circ, 60^\circ\}$. The SSM algorithm is used for DOA. The azimuth angle and the elevation angle have both been searched in the range of -90° to 90° ; Figure 4 and Figure 5 shows the resultant MUSIC power spectrum.

The proposed method was able to detect both Azimuth and Elevation angles correctly with estimation time=0.02s for 2D-MIMO compared to 0.06s estimation time in ULA-MIMO with 8 elements in [7] which represent an enhancement considering the increasing size of array. The simulation results are obtained using a PC with an Inter® Core™ i7-3520M CPU @ 2.90GHz and 4 GB RAM by running the MATLAB 2016a codes in the same environment. The effect of changing the initial phase of the transmitted signal was tested on the new proposed algorithm in DOA. Using only two signals received from two different mobile devices, where one signal has initial phase $=0$ while the other signal change in phase from 0 to π . Figure 6 and Figure 7 shows the estimation of elevation and azimuth angles of two targets (-2° with 0° initial phase and 2° with initial phase change from 0 to π). The results shows that whatever the initial phase

of the transmitted signal is, the new DOA method can estimate the angles of signal received from mobile devices correctly.

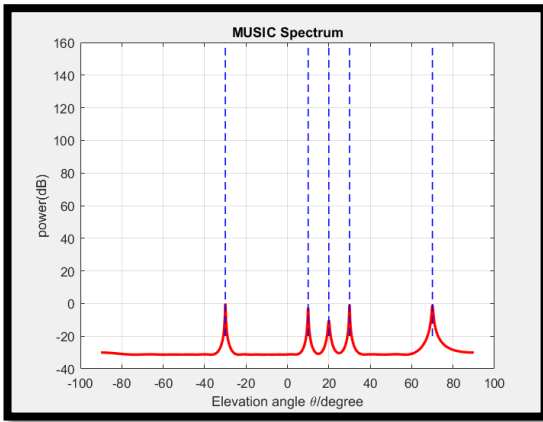


Figure 4. MUSIC spectrum of the proposed method (Azimuth angle)

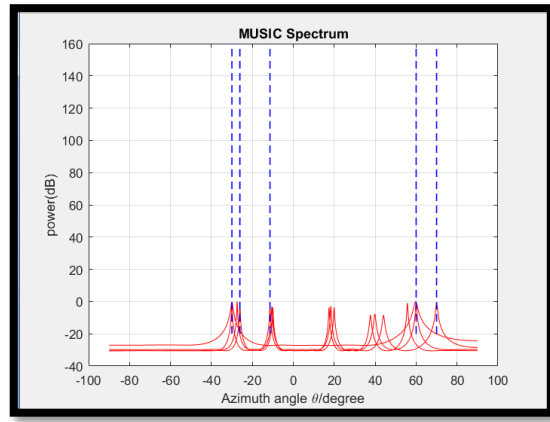


Figure 5. MUSIC spectrum of the proposed method (elevation angle)

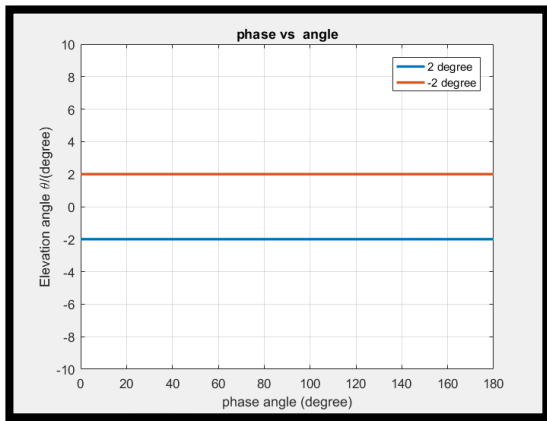


Figure 6. initial phase Vs estimated elevation angle in (degree)

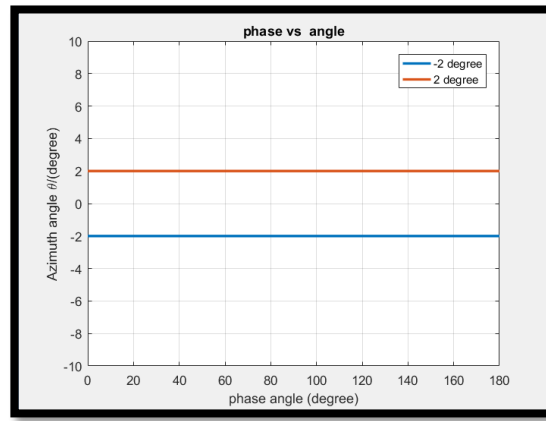


Figure 7. initial phase Vs estimated Azimuth angle in (degree)

A second test has been made to find the minimum separation in direction between two received signals, where the initial phase has no effect on detecting the two signals correctly. This distance found to be 3.14° (eg. From -1.57° to 1.57°) see Table 3. Since the beam width = $2 \cdot 0.89/N = 1.78 \cdot (180/\pi)/20 = 5^\circ$, this means the separation is less than beam width.

Table 3. Minimum Separation Distance M=20, N=20, Snap=100

Test No.	Angles range in (degree)	Max Initial phase angle in (degree)
1	-1.5, 1.5	177°
2	-1.57, 1.57	180°
3	-1.58, 1.58	180°

Figure 8 shows the two targets with 3.14° distance between them. When testing the effect of initial phase with targets separated by less than 3.14° , the DOA algorithm can't distinguish one from the other. The merging begins at initial phase 177° as shown in Figure 9.

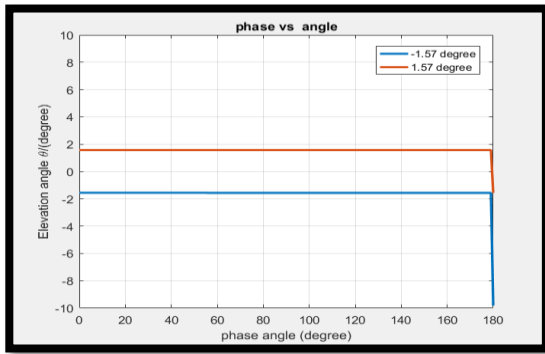


Figure 8. Minimum separation

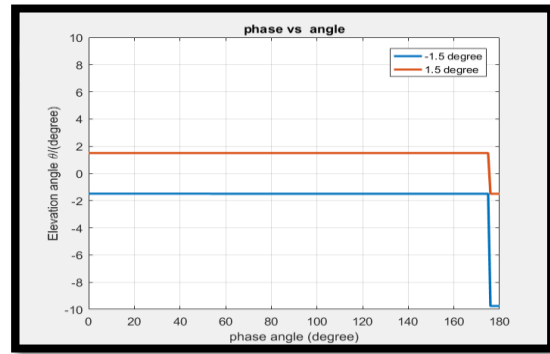


Figure 9. two targets with 3 degree separation

Third test was on the effect of initial phase with the number of snapshots taken from received signal as shown in Table 4. Figure 10 and Figure 11 shows the initial phase versus elevation angle different number of snapshots (40, and 60) respectively.

Table 4. effect of initial phase with the No. snapshots

Test No.	No. of Snapshots	Max Initial phase angle in (degree)
1	30	150°
2	40	160°
3	60	167°
4	100	168°

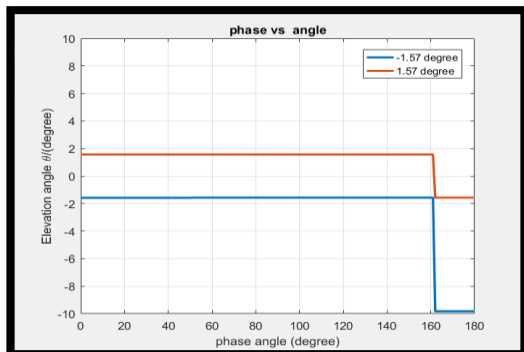


Figure 10. Phase Vs Elevation angle with No. snapshots=40

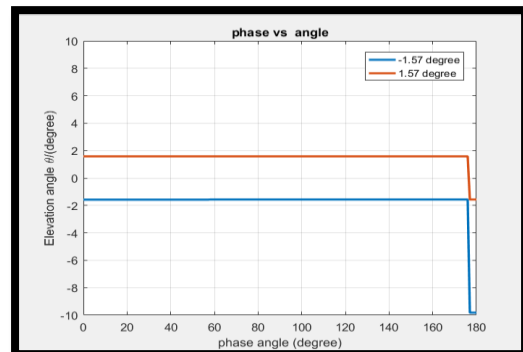


Figure 11. Phase Vs Elevation angle with No. snapshots=60

Test with snapshots less than 50 affects the ability to detect signals with the change of initial phase. The signals are detected as single target at initial phase 160° when the number of snapshots=40. New test was made on the effect of increasing the number of array elements in MMIMO on the required number of snapshots. Figure 12, Taking M=N=128 with snapshot number =10. The two targets taken to have separation between them 0.6° (less than beam width=1.78*(180/π)/128=0.79°) and with number of snapshots less than 50 it was able to detect them correctly with initial phase up to 167°.

Further increase to the number of elements of MMIMO (up to M=N=256) will support the same results, that with the increasing of the number of elements in MMIMO it can detect targets with separation less than beam width (BW=1.78*(180/π)/256=0.39°>0.3) and with minimum number of snapshots (e.g. 5 snapshots). As shown in Figure 13, with number of elements=5.

Table 5 contain sample of the effect of initial phase with different No. of elements and No of snapshots (targets at -3° , 3°).

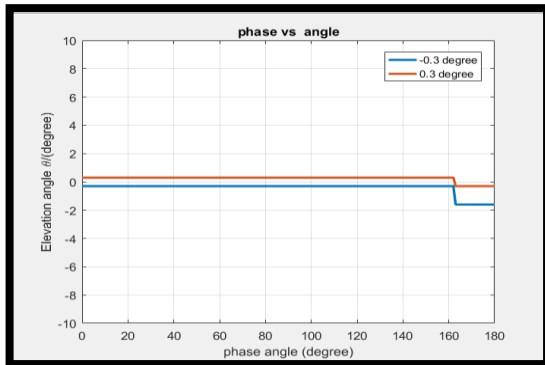


Figure 12. Phase Vs Elevation angle with $M=N=128$ and snapshots=10

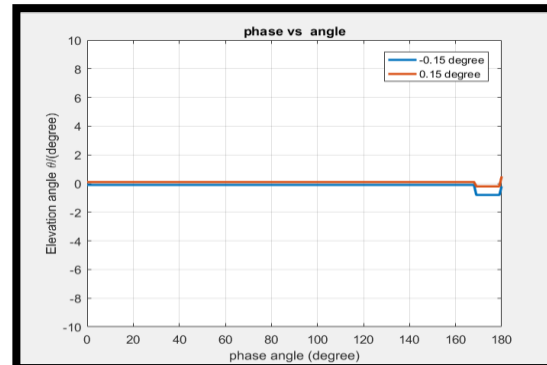


Figure 13. Phase Vs Elevation angle with $M=N=256$ and snapshots=5

Table 5. effect of initial phase with different No. of elements and No of snapshots (targets at -3° , 3°)

Test No.	No. Elements (M=N)	No.of Snap	Max Initial phase angle in (deg)
1	20	20	177°
2	20	10	143°
3	20	5	127°
4	128	20	180°
5	128	10	175°
6	128	5	155°
7	256	20	180°
8	256	10	178°
9	256	5	173°

6. Conclusion

In this paper the estimation of Azimuth and Elevation angles is done on FD-Massive MIMO. MUSIC algorithm has a sharp resolution for desired angle of interest. The traditional use of MUSIC algorithm for DOA estimation has been improved by reducing the computation operations and time. The proposed (SSM) method shows enhancement to the traditional method in two aspects. First aspect, reducing the complexity of calculating the covariance matrix by reducing its size, and second reducing the amount of time required to estimate the DOA. SSM method has proven these results through several tests. These tests compare SSM method with previously used methods, such as traditional MUSIC in [2], PM method using ESPRIT in [5], and improved PM in [6].

Another factor was tested in this paper, is the effect of initial phase of transmitted signal which found to have some effects on the DOA estimation. The proposed method was able to detect targets that are close to each other within less than beam width and initial phase π . While with decreasing the number of snapshots, the estimation is affected by initial phase. Meanwhile, the increasing of the number of elements in massive MIMO array reduces this effect, even when reducing the separation angle between sources to be less than beam width.

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