

Recursive Detection of M-Ary Signals over Fast Varying Mobile Communication Channel

Ahmed El-Sayed El-Mahdy

Department of Communications, Faculty of Information Engineering
& Technology, German University in Cairo, Egypt
Tel:+20227595565, Fax:+20227581041
ahmed.elmahdy@guc.edu.eg

Abstrak

Radio bergerak dikarakteristikan oleh kanal yang bervariasi cepat (fast time varying). Detektor konvensional yang didesain optimal untuk kanal non fading memiliki unjuk kerja terbatas saat diterapkan di kanal bervariasi cepat. Tulisan ini mengenalkan detektor rekursif untuk sinyal M-ary di kanal komunikasi fast time varying. Detektor yang diusulkan ini secara terus-menerus mengestimasi kanal secara langsung dalam perhitungan metrik dari fungsi log likelihood secara rekursif. Estimasi kanal dilakukan oleh bentuk kovarian dari pendekatan least square rekursif. Unjuk kerja detektor dievaluasi dalam bentuk probabilitas salah deteksi. Pengaruh dari pewaktuan dan offset fasa terhadap unjuk kerja detektor dievaluasi menggunakan simulasi. Hasil simulasi menunjukkan detektor yang diusulkan ini mampu mengakomodasi kanal bervariasi cepat (fast time varying) dengan unjuk kerja yang memadai.

Kata kunci : deteksi fast fading, rekursif

Abstract

Mobile radio is characterized by a fast time varying channel. Conventional detectors which designed optimal for non-fading channel exhibit a limited performance in fast time varying channel. In this paper a recursive detector for M-ary signals over fast time varying mobile communication channel is introduced. The proposed detector continuously estimates the channel directly within the metric calculation of the log-likelihood function in a recursive manner. The estimation of the channel is performed by the covariance form of the recursive least square approach. The performance of the detector is evaluated in terms of the misdetection probability. The effects of timing and phase offsets on the performance of detector are examined by simulation. Simulation results show that the proposed detector can accommodate the fast time varying channel with adequate performance.

Keywords: fast fading, recursive detection

1. Introduction

Multipath fading has a major effect on the performance of mobile communication systems. The time varying nature of the mobile wireless channel causes a limitation in the performance of the designed for non-fading channel even at high signal to noise ratio. Then, the development of mobile cellular resulted in increasing interest in the study of signal detection in the presence of this rapidly time varying environment. If the channel exhibits intersymbol interference (ISI) in the received data, adaptive equalizers [1-3], that use adaptive algorithms [4], are used to recover the received signal and to remove the ISI. When the channel is rapidly varying, tracking these variations becomes a challenging problem and hence the equalizer may not be able to track the channel.

Several articles discuss the problem of detection of M-ary signals [5-8]. In [5] the authors derived an exact expression for the symbol error probability (SEP) for coherent detection of M-ary PSK signals using array of antennas with optimum combining. They consider a Rayleigh flat fading channel. In [6], a low complexity breadth first tree detector, termed improved M-algorithm (IMA) is proposed. The authors showed that IMA works well without an energy compacting front end prefilter (FEP) even in frequency-selective channels. Moreover, the authors propose to make use of the cepstrum to compute the FEP via a minimum phase target impulse response. In [7], the effect of power and rate adaptation on the spectral efficiency of orthogonal frequency division multiplexing (OFDM) systems using M-ary quadrature

amplitude modulation (MQAM) is investigated in the presence of frequency selective and very rapidly time-varying fading channels, under power and instantaneous bit error rate (BER) constraints. Lower bounds on the maximum spectral efficiency of adaptive OFDM/MQAM systems with perfect and imperfect channel state information (CSI) are obtained, together with a closed-form expression for the average spectral efficiency of adaptive OFDM systems. In [8], the authors developed a computationally efficient algorithm for the Maximum Likelihood (ML) sequences estimation (MLSE) of an M-ary Phase Shift keying (M-PSK) signal transmitted over a frequency non-selective slow fading channel with an unknown complex amplitude and an unknown variance additive white Gaussian noise. The proposed algorithm also provides the ML estimates of the complex amplitude and the noise variance that are critical in signal activity detection and demodulation in the modern cognitive communication receivers.

In this paper, such rapidly fading channels are considered, specifically, the channel model described in [9, 10, 11, 12] is used. In this model, the time varying channel taps are modeled by a finite linear combination of complex exponentials. The basis expansion approach of [9, 11, 12] is used in this paper and the time varying channel coefficients are expanded into a set of basis sequences and expansion parameters. These basis sequences are assumed to be known while the expansion parameters are unknown and needs to be identified. In [9], higher order statistics are used to estimate the expansion parameters based on minimizing a moment matching criterion and use them to estimate the time varying channel coefficients. It is proved that identifiability of the channel cannot be achieved from the second order cumulants. It requires fourth order cumulant to identify it under the assumption of linear independence on the basis sequences.

In this paper, an adaptive detector structure for detection of M-ary signals over a rapidly time varying fading channel is presented. The detector is based on maximum likelihood criterion. The channel impulse response is expanded onto a set of basis sequences and a time invariant (TI) expansion parameters. The proposed detector continuously estimates the time invariant expansion parameters directly within the metric calculation of the log-likelihood function. The recursive least square (RLS) approach is used to perform this estimation. The performance of the detector is demonstrated in the simulation section. Also, the effects of timing and phase offsets are examined by simulation.

The paper is organized as follows. In Section 2, the proposed detector is derived including the system model, recursive evaluation of the ML metric, channel parameter estimation and the operation of the proposed detector. In Section 3, the results of the computer simulation to demonstrate the performance of the detector are provided. Conclusions are provided in Section 4.

2. The Proposed Detector

2.1. System Model

A data sequence is transmitted over a rapidly time varying channel using one out of M signals. The sampled signals at time instant k is denoted by $x_i(k)$; $i = 1, 2, \dots, M$. The discrete equivalent (or combined) impulse response of the channel including transmitter filter is denoted by $v(k, l)$ where l represents the channel memory ($l = 0, 1, \dots, L-1$). The unknown channel taps are correlated even if the scatters in the physical channel are uncorrelated. The additive noise is assumed to be white Gaussian noise. The received signal is filtered and sampled at a rate of f_s . The problem is formulated as follows. Given the discrete received signal $z(k)$, the detector must decide which signal $x_i(k)$ has been sent where $x_i(k)$ is the sampled transmitted signal. That is, the hypotheses H_i ; $i = 1, 2, \dots, M$, over the observation interval N_s , are given by

$$H_i: z(k) = \sum_{l=0}^{L-1} x_i(k-l)v(l, k) + w(k), \quad (1)$$

where L is the channel memory length and $w(k)$ are independent and identically distributed

(i.i.d) complex valued zero mean Gaussian noise samples with known variance σ_n^2 . The optimum maximum likelihood (ML) detector chooses the hypothesis H_i with the largest likelihood function, but it requires perfect knowledge of the channel time varying coefficients, $v(k, l)$. These coefficients are usually modeled as Gaussian random process. However, a more precise description of the time variations of the channel coefficients can be provided for the multipath channels, which have small number of reflectors. For example, for constant vehicle velocity, the mobile radio channel is almost periodically varying when the multipath delays change linearly with time due to the carrier modulation inherent in the transmitted signal [9]. Its time varying coefficients can be expressed as a combination of exponentials whose frequency depends on the carrier frequency and the vehicle speed [11]. In this paper, we consider the channels, which their time varying coefficients $v(k, l)$ can be approximated by a linear combination of a finite number of basis sequences $f_n(k)$:

$$v(l, k) = \sum_{n=1}^N \theta_{nl} f_n(k) \quad (2)$$

where θ_{nl} are non-random expansion parameters and $f_n(k)$ are basis sequences. For fast mobile radio channels, these basis sequences are expressed as [9, 12]:

$$f_n(k) = \exp\{j\alpha_n k\} \quad (3)$$

where α_n are some frequencies; $n = 1, 2, \dots, N$. These frequencies are assumed to be known and estimation of these frequencies are found in [9]. Practical values are used for these basis sequences in the simulation section.

2.2. Recursive Evaluation of the ML Metric

In this subsection, the log-likelihood metric for the received signal is derived and evaluated recursively. Using (1), (2), and (3), $z(k)$ can be expressed as:

$$z(k) = \sum_{l=0}^{L-1} \sum_{n=1}^N \theta_{nl} x_i(k-l) f_n(k) + w(k), \quad (4)$$

$$i = 1, 2, \dots, M$$

Let us define the following vectors and matrices:

$$\Theta_l = [\theta_{1l}, \theta_{2l}, \dots, \theta_{Nl}]^T \quad (5)$$

and

$$\mathbf{X}_{l,k}^{(i)} = [f_1(k) x_i(k-l) \quad f_2(k) x_i(k-l) \quad \dots \quad f_N(k) x_i(k-l)] \quad (6)$$

where i is referred to the hypothesized signal. Let the coefficients Θ_l be assembled into the $(N \times L) \times 1$ unknown vector Ψ :

$$\Psi = [\Theta_0^T \quad \Theta_1^T \quad \dots \quad \Theta_{L-1}^T]^T \quad (7)$$

and also

$$\mathbf{X}_k^{(i)} = [\mathbf{X}_{0,k}^{(i)} \ \mathbf{X}_{1,k}^{(i)} \ \dots \ \mathbf{X}_{L-1,k}^{(i)}] \quad (8)$$

where the superscript T' denotes matrix transposition. Using the above definitions, we can rewrite (4) in the following representation;

$$z(k) = \mathbf{X}_k^{(i)} \boldsymbol{\Psi} + w(k) \quad (9)$$

Let $\mathbf{z} = [z(1), z(2), \dots, z(N_s)]^{T'}$ denotes the noisy received signal vector and $\mathbf{x}_i = [x_i(1), x_i(2), \dots, x_i(N_s)]^{T'}$ denotes the i^{th} transmitted signal vector. Since the observation noise is assumed to be white Gaussian, then the probability density function (PDF) of the received signal vector \mathbf{z} under hypothesis H_i , conditioned on the channel parameters vector $\boldsymbol{\Psi}$, can be written as:

$$f(H_i, \mathbf{z} / \boldsymbol{\Psi}) = \frac{1}{(\pi\sigma_n^2)^{N_s}} \exp \left\{ \frac{-1}{\sigma_n^2} \sum_{k=1}^{N_s} |z(k) - \mathbf{X}_k^{(i)} \boldsymbol{\Psi}|^2 \right\} \quad (10)$$

For equi-probable messages, the relevant conditional log-likelihood function (LLF) under hypothesis H_i may be written as:

$$\Delta_{N_s}(H_i; \mathbf{z} / \boldsymbol{\Psi}) = \sum_{k=1}^{N_s} |z(k) - \mathbf{X}_k^{(i)} \boldsymbol{\Psi}|^2; \quad i = 1, \dots, M \quad (11)$$

For a given $\boldsymbol{\Psi}$, the optimum ML detector chooses the hypothesis \hat{H} that maximizes $f(H_i; \mathbf{z} / \boldsymbol{\Psi})$ or, equivalently, minimizes $\Delta_{N_s}(H_i; \mathbf{z} / \boldsymbol{\Psi})$ that is:

$$\hat{H} = H_i : \Delta_{N_s}(H_i; \mathbf{z} / \boldsymbol{\Psi}) < \min_{q \neq i} \Delta_{N_s}(H_q; \mathbf{z} / \boldsymbol{\Psi}) \quad (12)$$

An estimate of $\boldsymbol{\Psi}$ is required to evaluate (12). The evaluation of (12) and the estimation of $\boldsymbol{\Psi}$ can be performed recursively in time as follows: Let $\hat{\boldsymbol{\Psi}}_k^{(i)}$ denotes the estimation of the channel parameters vector under hypothesis i and at time step k . Then by substitution of $\hat{\boldsymbol{\Psi}}_k^{(i)}$ into (11), we have the log-likelihood function $\Delta_{N_s}(H_i; \mathbf{z} / \hat{\boldsymbol{\Psi}}_k^{(i)})$. In fact, if the equation:

$$\Delta_m(H_i; \mathbf{z} / \hat{\boldsymbol{\Psi}}_k^{(i)}) = \sum_{k=1}^m |z(k) - \mathbf{X}_k^{(i)} \hat{\boldsymbol{\Psi}}_k^{(i)}|^2 \quad (13)$$

is defined for $m = 1, 2, \dots, N_s$, we easily obtain the recursion

$$\Delta_m(H_i; \mathbf{z} / \hat{\boldsymbol{\Psi}}_k^{(i)}) = \Delta_{m-1}(H_i; \mathbf{z} / \hat{\boldsymbol{\Psi}}_k^{(i)}) + |r(m) - \mathbf{X}_m^{(i)} \hat{\boldsymbol{\Psi}}_m^{(i)}|^2 \quad (14)$$

where $\Delta_{m-1}(\mathbf{z} / \hat{\boldsymbol{\Psi}}_k^{(i)})$ represents the evaluation of the LLF within the interval $k = 1, 2, \dots, m-1$. This formula suggests a recursive solution for the minimization of (11). A separate channel

parameters vector estimate $\hat{\Psi}_k^{(i)}$ is created for each hypothesis using the observations and the hypothesized signal.

2.3. Estimation of the Channel Parameters Vector

Possible approaches to a sample-by-sample estimation of the channel parameters vector are gradient based methods (like LMS) and recursive least square based methods. It is important to observe that the true channel parameters vector is time invariant, so the task of the adaptive algorithm in the proposed approach is to converge to the channel parameters as opposed to tracking them. The covariance form of the least square (CRLS) approach [12] is chosen to perform this estimation due to its fast convergence. Applying this form results in the following algorithm:

$$\hat{\Psi}_k^{(i)} = \hat{\Psi}_{k-1}^{(i)} + \mathbf{K}_k^{(i)} [z(k) - \mathbf{h}_k^{(i)T'} \hat{\Psi}_{k-1}^{(i)}] \quad (15)$$

where

$$\mathbf{K}_k^{(i)} = \mathbf{P}_{k-1}^{(i)} \mathbf{h}_k^{(i)} \left[\mathbf{h}_k^{(i)T'} \mathbf{P}_{k-1}^{(i)} \mathbf{h}_k^{(i)} + \frac{1}{\mu_k} \right]^{-1} \quad (16)$$

is the weighting vector and,

$$\mathbf{P}_k^{(i)} = \left(\mathbf{I} - \mathbf{K}_k^{(i)} \mathbf{h}_k^{(i)T'} \right) \mathbf{P}_{k-1}^{(i)} \quad (17)$$

is the estimation error covariance matrix. The factor μ_k is a weighting factor, which is set to one for equal weights. It is noted that, $\hat{\Psi}_k^{(i)}$ is the estimate of the channel parameters vector at time k under hypothesis i and the term $\mathbf{h}_k^{(i)T'} \hat{\Psi}_{k-1}^{(i)}$ in (15) is a prediction of the actual measurement $z(k)$ under hypothesis i . The above algorithm updates $\hat{\Psi}_k^{(i)}$ by iteratively adding an adjustment term. The adjustment term is given by a vector of weights $\mathbf{K}_k^{(i)}$, which is multiplied by the error $[z(k) - \mathbf{h}_k^{(i)T'} \hat{\Psi}_{k-1}^{(i)}]$ to determine the parameter change. The value of this error is small when there is a matching between the observation $z(k)$ and the term $\mathbf{h}_k^{(i)T'} \hat{\Psi}_{k-1}^{(i)}$ (i.e. when the observation $z(k)$ contains the correct hypothesis) and it is large when there is a mismatch between them. Since the CPV is time invariant, the estimation algorithm is initialized using an initial guess $\hat{\Psi}_0^{(i)}$ for the CPV or using $\hat{\Psi}_0^{(i)} = 0$. The initial value for $\mathbf{P}_0^{(i)-1}$ is given by: $\mathbf{P}_0^{(i)-1} = \varepsilon^{-1} \mathbf{I}_{N \times L}$, where ε is a large positive constant (typical value: 10^2) and $\mathbf{I}_{N \times L}$ is the identity matrix with dimension $(N \times L) \times (N \times L)$.

2.4. Operation of the Proposed Detector

The structure of the detector is shown in Figure.1. As explained above, the decision of the detector is based on the evaluation of the function, $q_i = \Delta_{N_s}(H_i; \mathbf{z} / \hat{\Psi}_k^{(i)})$, then the detector executes the following algorithm for each hypothesis $i = 1, 2, \dots, M$:

- (i) Start with an initial estimate for the channel parameters vector $\hat{\Psi}_0^{(i)}$.
- (ii) Use the observation and the hypothesized signal to find $\hat{\Psi}_1^{(i)}$.
- (iii) Substitute $\hat{\Psi}_1^{(i)}$ in (13) and then find $\Delta_1(H_i; \mathbf{z} / \hat{\Psi}_1^{(i)})$.

- (iv) Update the estimate of the channel parameters vector to find $\hat{\Psi}_2^{(i)}$ and then use it to evaluate $\Delta_2(H_i; \mathbf{z} / \hat{\Psi}_2^{(i)})$ using (14).
- (v) Repeat steps (2) to (4) until all the data samples have been processed (i.e. when $k = N_s$) and then obtain q_i .
- (vi) Compare among $q_i; i=1, 2, \dots, M$ and determine i that corresponds to the minimum value of q .

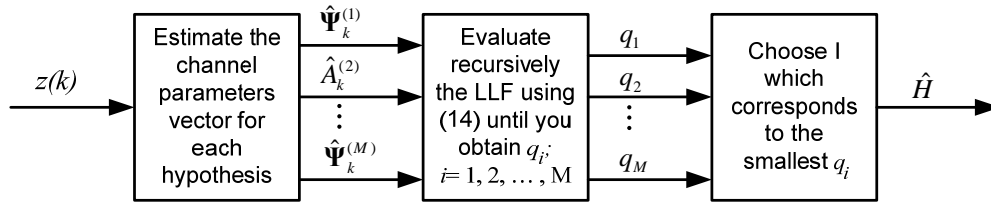


Figure 1. Structure of the proposed adaptive detector for the rapidly time varying fading channel

3. Computer Simulations and Results

In this section, the performance of the detector is evaluated for the mobile radio-fading channel defined in (2) using Mont Carlo simulation. The channel has two time varying taps, each one is a linear combination of three basis sequences denoted by $f_{1,k} = 1$, $f_{2,k} = \exp\{j\pi k / 60\}$, and $f_{3,k} = \exp\{j\pi k / 100\}$. These bases simulate a realistic situation for a 900 MHz carrier frequency, bit rate around 20 Kb/s and a vehicle speed of 100 Km/h. The values of expansion parameters θ_{nl} are given in Table I [9] are chosen so that the fading channel passes through the minimum and non-minimum phase regions.

Table 1. Basis Expansion Parameters [5]

θ_{nl}	$n=1$	$n=2$	$n=3$
$l=0$	1	j	2
$l=1$	1	0.5	$-j$

The input to the channel is a QPSK signal. The generated signal has independent and identically distributed symbols. The frame length is 512 samples. A white Gaussian noise is simulated and added to the signal at the input of the detector. The signal to noise ratio is defined as $SNR=10\log(E_b/N_o)$ where $N_o/2$ is the noise power spectral density and E_b is the energy per bit. It is assumed that no phase and time offsets in the carrier. However, the effects of these offsets are studied in this section.

The performance comparison among the proposed detector, the effect of model mismatch and the reference detector is evaluated in terms of misdetection probability versus signal to noise ratio (SNR) at the input of the detector. The results of comparison are shown in Figure.2. This figure shows that the proposed detector has adequate performance and allows an effective tracking of the channel. The lower curve in Figure.2 represents the unrealistic case where the channel is assumed to be known to the detector. Hence, this curve can be considered as a lower bound or a reference detector for comparison purposes of the other detectors.

Figure 2 also shows that the degradation in the misdetection probability that results when an incorrect channel model is adopted. In this case, a random disturbance is added to the

real and imaginary parts of the channel time varying coefficients $v(l, k) = \sum_{n=1}^N \theta_{nl} f_n(k) + e_l(k)$.

The disturbance $e_l(k)$ is generated as an i.i.d. Gaussian random variables with standard deviation 0.2. Then, the proposed algorithm is applied and the misdetection probability is evaluated. The simulated results indicate that the anticipated increase in misdetection probability under this mismatch condition is actually not significant at low SNR. This is because the additive noise dominated the performance at low range of SNR. As SNR increases, the influence of the additive noise decreases, and the model mis-match becomes the dominant source of degradation, causing deterioration in misdetection probability.

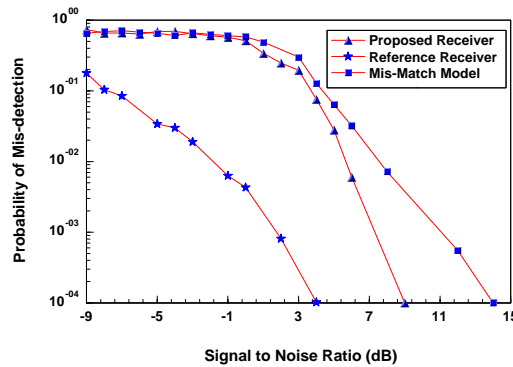


Figure 2. Probability of misdetection for different detectors

Figures 3 and 4 demonstrate the effects of timing and phase offsets on the performance of the detector respectively. The figures are plotted for SNR=0, 5, and 9 dB. These figures show that the detector is able to detect the signal reliably when phase or timing offset is small. When phase or timing offset increases, the detector performance degrades rapidly. The reason for this degradation is that: the increase in these offsets causes increase in the residual error in estimation of the channel parameters vector Ψ_k (since the estimation of Ψ_k depends on the observation which is shifted in time or phase) and this introduces an error in the log-likelihood metric calculation given by (13), accordingly, the misdetection probability degrades rapidly. Figures 3 and 4 also show that, there is a range in which the effect of phase or time offset can be neglected and the misdetection probability in this range is small. This range is shown in these figures for SNR=9 dB. This range depends on the SNR (it increases as the SNR increases) and it is up to $t_o/T = 0.4$ for the timing offset and 0.5 rad. for phase offset, where T is the symbol duration.

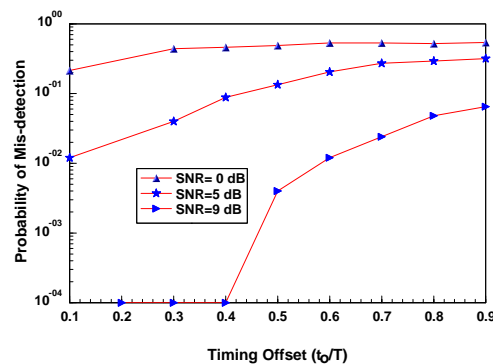


Figure 3. Effect of timing offset on the performance of the detector for different signal to noise ratios

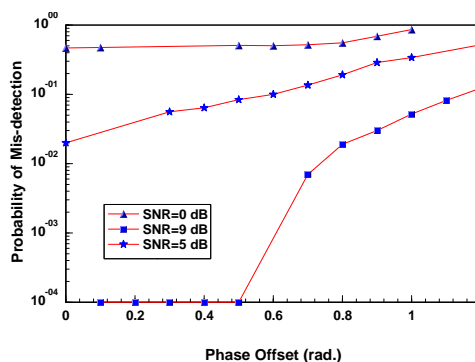


Figure 4. Effect of phase offset on the performance of the detector for different signal to noise ratios

4. Conclusion

In this paper, a recursive detector for detecting M-ary signals over a rapidly time varying mobile communication channel has been presented. The channel has been expanded into a set of basis sequences and time invariant expansion parameters. This expansion idea provides a helpful tool for addressing problems of such channels. The time invariant expansion parameters of the channel have been estimated continuously and directly within the metric calculation of the log-likelihood function in a recursive manner. The performance of the detector has been evaluated in terms of the misdetection probability. The detector provides adequate performance for this type of channel. Also, the effects of timing and phase offsets on the performance of detector have been studied.

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