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Tree Physiology Optimization in Constrained Optimization Problem

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Abstract

Metaheuristic algorithms are proven to be more effective on finding global optimum in numerous problems including the constrained optimization area. The algorithms have the capacity to prevail over many deficiencies in conventional algorithms. Besides of good quality of performance, some metaheuristic algorithms have limitations that may deteriorate by certain degree of difficulties especially in real-world application. Most of the real-world problems consist of constrained problem that is significantly important in modern engineering design and must be considered in order to perform any optimization task. Therefore, it is essential to compare the performance of the algorithm in diverse level of difficulties in constrained region. This paper introduces Tree Physiology Optimization (TPO) algorithm for solving constrained optimization problem and compares the performance with other existing metaheuristic algorithms. The constrained problems that are included in the comparison are three engineering design and nonlinear mathematic problems. The difficulties of each proposed problem are the function complexity, number of constraints, and dimension of variables. The performance measure of each algorithm is the statistical results of finding the global optimum and the convergence towards global optimum.

Keywords: *metaheuristic algorithm, constrained problem, engineering design, tree physiology optimization, global optimum*

1. Introduction

Metaheuristic optimization is a well known algorithm of solving numerous optimization problems including engineering fields. The ability of searching to near optimal solution within a reasonable time frame made the algorithms as significant methods as proven in numerous literatures. The major properties that made the algorithm a successful method lies in two mechanisms: diversification and intensification [1]. Diversification is refers to the ability of the algorithm to explore in the search space. The main objective of diversification is to ensure that all possible solutions are verified, thus the probability of finding near optimal solution is higher. Alternatively, intensification or exploitation refers to the ability of searching within confined area or also denoted as a local search. The objective of intensification is to focus on the search in local region by exploiting the information of current good solution found in that region. A good metaheuristic algorithm has dynamic balance between both components [1].

Most of the algorithms are inspired by nature and has certain tradeoff between exploration and exploitation strategy. Several algorithms discussed in this paper is summarised in Table 1. Nonetheless, there are some limitations of each algorithm to certain problem type [2]. Based on ‘no-free lunch theorem’, there are no such algorithms that are able to solve all of the problem type effectively [3]-[4]. Some algorithm may be capable to find an optimized solution of a specific problem that other algorithm may not. For this reason, an analysis and comparison of metaheuristic algorithms performance on a specific constrained optimization problem is discussed. The constrained optimization problem refered to in this paper consists of the benchmark engineering design problem and nonlinear mathematical problem. The engineering design problem is a benchmark problem of miscellaneous engineering issues such as constraint of energy resources, demand of lightweight, efficient and low cost structure. Such problem has become significantly important in modern engineering design [5]. This paper proposes a metaheuristic algorithm denoted as Tree Physiology Optimization (TPO) as an effective algorithm with faster convergence towards global optimum solution. This paper also

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compares the performance of TPO with other metaheuristic algorithms with constrained optimization problem. An introduction of TPO as metaheuristic algorithm for constrained optimization problem is introduced in Section 2. In Section 3, a short overview of other metaheuristic algorithms and the proposed constrained problem is presented. Section 4 compares the efficiency of each metaheuristic with the proposed problem and the last section summarized a conclusion for this paper.

2. Tree Physiology Optimization

The Tree Physiology Optimization (TPO) algorithm is enthused form plant growth system [6]. The idea of TPO consists of two main components, which are: shoots- and roots growth. The shoots growth of any plant is driven by the light intensity as positive phototropism behaviour [7]. The plant shoots extend towards light in order to convert water and carbon dioxide (CO2) into carbon. Carbon is an essential source for plant especially for root growth. The propagation of the shoots is depending on the nutrients supplied by the roots system. Contrary, the roots counterparts consume carbon gained by the shoots sytem and grow towards soils as positive gravitropism behaviour in order to search for nutrients [8]. The shoot-root relationship is simplified as a Thornley-model [8-9]. Based on the model, shoots consumed nutrients and extend towards light for photosynthesis process and produces carbon, whereas roots consumed carbon gained from shoots system and elongate towards soil for nutrient absorption. This idea inspired an optimization process, which is refered as Tree Physiology Optimization [6]. The TPO algorithm is established with four equations that represents shoots extension, carbon gain, root elongation, and nutrient absorption. The shoots extension is defined as:

$$S_i^{kj} = S_i^{kj} + \left(S_{gbest} - S_i^{kj}\right) + \beta N_i^{kj} \quad (1)$$

With $S_i^{kj}$is the value of current shoot during ith iteration, of kth leafs and jth branches, S_{gbest} is equivalent to global best value from all branches, N_i^{kj} is the value of nutrient initiated by root, and β is a diversification constant. Higher β lead to more diversified shoots. Too big β leads to dispersed shoots and may take longer time to converge, whereas too small β might lead to less scattered of shoots allocation and thus resulted in local optimum. Each shoot undergo photosynthesis and converts into carbon gain. The value of carbon-gain corresponds to the deviation of individual shoot with its branch best as:

$$C_i^{kj} = \theta^i \left(S_{popbest} - S_i^{kj}\right) \quad (2)$$

C_i^{kj} is current carbon gain, $S_{popbest}$ is best shoot of current branch. The value θ is equivakent to a power-law so as to reduce the randomness as iteration increases in a pattern of a monotonic decreasing function. Typical value for better convergence is 0.9. C_i amplifies the root elongation in search for more nutrients as in (3). Therefore a good θ value lead to good amplification of roots distribution.

$$r_i^{kj} = r_i^{kj} + \alpha\, C_i^{kj}\, \epsilon \quad (3)$$

r_i^{kj} is equivalent to current root, α is an absorption parameter, ε is a random numbers. The root elongates into soil with a random motion. The value of α has the same objective as β in shoot extension: to ensure a better diversification and convergence. The nutrient uptake is assumed as a factor of root elongation.

$$N_i^{kj} = \theta\left(r_i^{kj} - r_{i0}^{kj}\right) \quad (4)$$

With r_i^{kj} and r_{i0}^{kj} as the current and previous value of root respectively. Some effort in using TPO as optimization tool are applied in numerous application such as nonlinear ANFIS modeling [6], neural network training [10], and PID tuning [11]. In nonlinear optimization problems, TPO outperformed other metaheuristic algortihm with lesser computation time [12].

3. Research Method

The performance of TPO is compared with other three metaheuristic algorithms in the constrained optimization problem. The overviews of other three metaheuristics are summarized in Table 1. The capability of proposed algorithms is verified with constrained optimization problem as shown in Table 2. These problems have different difficulties such as number of variables, number of constraints and dimension complexity.

Table 1. Overview of Four Metaheuristic Algorithms

Num	Algorithm (year)	Main features
1	Particle Swarm Optimization, PSO (1995)	Swarm-based :- inspired by swarm movement of creatures. Two type of solution attraction: local and global best value per-iteration with two equations, speed and position: $v_i^{t+1} = v_i^t + \alpha\epsilon_1(g_* - x_i^t) + \beta\epsilon_2(x_i^* - x_i^t)$. $x_i^{t+1} = x_i^t + v_i^{t+1}$ [13]
2	Firefly Algorithm, FA (2007)	Inspired from fireflies flashing mechanism. Solution equivalent to attrattiveness of flashing behaviour (fitness). Attraction of firefly i to firefly j with: $x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2}(x_j - x_i) + \alpha\epsilon$ with distance between firefly i and j: $r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ [14]
3	Cuckoo Search, CS (2009)	Inspired from brood parasitism behaviour of cuckoo bird. New solution is generated via levy flight: $x_i^{t+1} = x_i^t + \alpha\, L$ with L as levy flight. The fitness is correlated to randomly choosen best nest. A fraction of worst nest is eliminated by a probability factor [15]
4	Tree Physiology Optimization, TPO (2013)	Inspired from physiological concept of plant growth. Shoots branches as potential solution and root's growth amplify the search. Search process governed by (1)–(4).

Table 2 indicates the most widely used constrained optimization problems with three engineering design problems and one nonlinear mathematical problem.

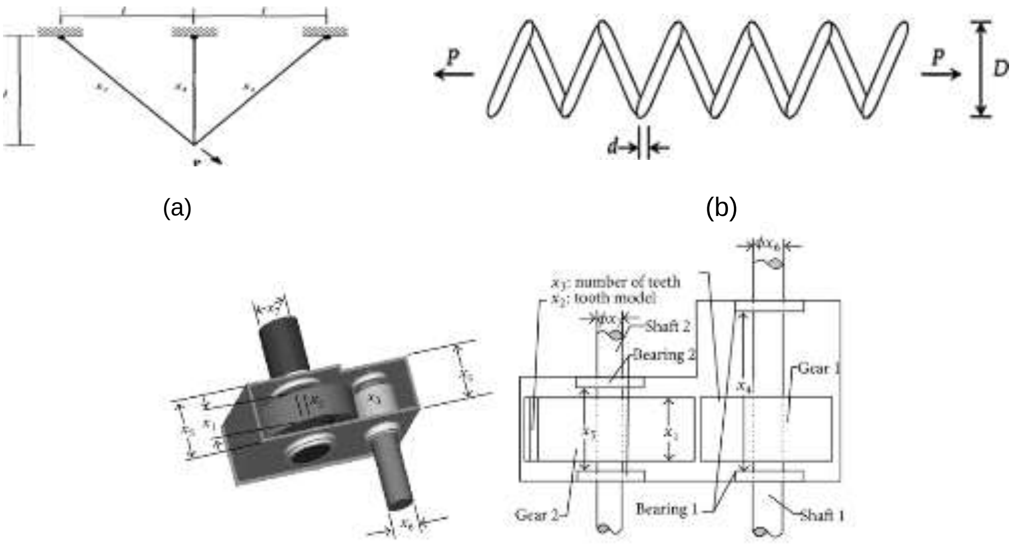
Table 2. Characteristics of constrained problems

Num	Constr. Prob.	Equations
F1	Three-bar truss [16] (2 dimensions.) (2 constraints.)	$\min f(x) = \left(2\sqrt{2x_1} + x_2\right) l$; Constraints: $g_1(x) = \frac{\sqrt{2x_1} + x_2}{\sqrt{2x_1^2 + 2x_1x_2}} P - \sigma \le 0$ $g_2(x) = \frac{x_2}{\sqrt{2x_1^2 + 2x_1x_2}} P - \sigma \le 0$ $g_3(x) = \frac{1}{\sqrt{2x_2 + x_1}} P - \sigma \le 0;\ 0 \le x_1 \le 1, i = 1,2$
F2	Spring design [17] (3 dimensions.) (4 constraints.)	$\min f(x) = (x_3 + 2)\, x_2 x_1^2$; Constraints: $g_1(x) = 1 - \frac{x_2^3 x_3}{71.785x_1^4} \le 0$ $g_2(x) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} \le 0$ $g_3(x) = 1 - \frac{140.45x_1}{x_2^2 x_3} \le 0$ $g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \le 0$ $0.05 \le x_1 \le 2;\ 0.25 \le x_2 \le 1.3;\ 2 \le x_3 \le 15$
F3	Speed reducer [16,17] (7 dimensions) (11 constraints)	$\min f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$; Constraints: $g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \le 0$ $g_2(x) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \le 0$ $g_3(x) = \frac{1.93x_4^3}{x_2x_6^4x_3} - 1 \le 0$ $g_4(x) = \frac{1.93x_5^3}{x_2x_7^4x_3} - 1 \le 0$ $g_5(x) = \frac{\sqrt{\left[(745(x_4/x_2x_3))^2 + 16.9e6\right]}}{110x_6^3} - 1 \le 0$ $g_6(x) = \frac{\sqrt{\left[(745(x_5/x_2x_3))^2 + 157.5e6\right]}}{85x_7^3} - 1 \le 0$ $g_7(x) = \frac{x_2x_3}{40} - 1 \le 0$ $g_8(x) = \frac{5x_2}{x_1} - 1 \le 0$ $g_9(x) = \frac{x_1}{12x_2} - 1 \le 0$ $g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0$ $g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0$ $2.6 \le x_1 \le 3.6, 0.7 \le x_2 \le 0.8, 17 \le x_3 \le 28, 7.3 \le x_i \le 8.3\ for\ i = 4,5,$ $12.9 \le x_6 \le 3.9, 5.0 \le x_7 \le 5.5$
F4	Himmelblau [18] (5 dimensions.) (6 constraints)	$\min f(x) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141;$ Constraints: $g_1(x) = 85.334407 + 0.0056858x_2x_5 + 2.6^{-4}x_1x_4 - 2.2053^{-3}x_3x_5 - 92 \le 0$ $g_2(x) = -85.334407 - 5.6856^{-3}x_2x_5 - 2.6^{-4}x_1x_4 + 2.2053^{-3}x_3x_5 \le 0$ $g_3(x) = 80.51249 + 7.1317^{-3}x_2x_5 + 2.9955^{-3}x_1x_2 + 2.1813^{-3}x_3^2 - 110 \le 0$ $g_4(x) = -80.51249 - 7.1317^{-3}x_2x_5 - 2.9955^{-3}x_1x_2 - 2.1813^{-3}x_3^2 + 90 \le 0$ $g_5(x) = 9.300961 + 4.7026^{-3}x_3x_5 + 1.2547^{-3}x_1x_3 + 1.9085^{-3}x_3x_4 - 25 \le 0$ $g_6(x) = -9.300961 - 4.7026^{-3}x_3x_5 - 1.2547^{-3}x_1x_3 - 1.9085^{-3}x_3x_4 + 20 \le 0$ $78 \le x_1 \le 102, 33 \le x_2 \le 45, 27 \le x_i \le 45, i = 3,4,5$

The three engineering design problems for this benchmark are shown in Figure 1. The difficulties of each benchmark problem as tabulated in Table 2 are the function complexity, number of constraints, and dimension of variables. Each algorithm is set according to their nature of coding and searching mechanism as depicted in Table 3.

Table 3. Algorithm Setting

Algo.	Parameters	Algo.	Parameters
PSO	Iteration=2000	CS	Iteration=2000
	Population=500		Nests=25
	α=1.05; β=1.1		$p_a = 0.25$
FA	Iteration=2000	TPO	Iteration=2000
	Firefles=20		Leaves=500
	$\alpha = 0.25;\ \beta = 0.2;\ \gamma = 1$		Branch=20
			$\alpha = 0.3;\ \beta = 0.7;\ \theta = 0.01$



(a) (b)

(c)

Figure 1. Engineering design problems with (a): Three-bar truss [16], (b): Spring design problem [17] and (c) Golinski speed reducer problem [16], [17]

4. Results and Analysis

The comparison between each algorithm is executed by 2.6GHz computer processor. The simulation and statistical analysis are carried out using MATLAB and STATSGRAPHICS Centurion respectively. Each algorithm is executed ten times for each problem and the obtained results are evaluated. The focuses of evaluation include statistical results and convergence.

4.1. Statistical Comparison

The statistical results of each algorithm by every problem are tabulated in Table 4. The best result of each category is highlighted in bold. The values in Table 4 are divided by constrained problem (F), average, best value, worst value and standard deviation (σ). Based on the results, PSO has the lowest average and the best solution in F1 followed by CS and TPO. TPO has the lowest variation among the results of F1. In F2 optimization, TPO shows the best results for all categories followed by FA, PSO and CS for the mean value. In F3, CS has the lowest results for all categories followed by TPO. TPO also outperform other algorithms in F4 with lowest average and variation of the results. Overall, most of the best results are shown by TPO and CS. The advantage of TPO is due to the parallel search of leaves in each defined branches, which is equivalent to (leaves x branches) search agents. Thus the search space is broader within the constrained area.

Table 4.Statistical results of each algorithm

F	Algo.	Mean	Best	Worst	σ
F1	PSO	**263.743**	**263.054**	263.991	3.34E-01
	FA	263.897	263.896	263.898	5.68E-03
	CS	263.794	263.143	264	0.238263
	TPO	263.896	263.896	**263.896**	**2.13E-07**
F2	PSO	0.013072	0.012826	0.13671	2.35E-04
	FA	0.013067	0.012833	0.013282	1.61E-03
	CS	0.013504	0.012745	0.015615	8.77E-03
	TPO	**0.012736**	**0.012666**	**0.01281**	**4.60E-05**
F3	PSO	3042.62	3027.15	3069.53	1.44E+01
	FA	3015.48	3007.16	3035.89	9.73E+00
	CS	**2994.49**	**2994.47**	**2994.61**	**4.70E-02**
	TPO	2996.63	2995.05	2997.76	1.25E+00
F4	PSO	-30974.4	-31010.4	-30931.1	2.66E+01
	FA	-31020.2	-31025.4	-30977.3	1.51E+01
	CS	-31025	**-31026.6**	-31020.6	1.77E+00
	TPO	**-31025.2**	-31025.6	**-31022.2**	**1.07E+00**

4.2. Convergence Analysis

The convergence of each algorithm in single run is shown in Figure 2. The figure is divided into four charts that represent each constrained problem (F1–F4). Based on the figure, TPO has fastest convergence towards global optimum in all four constrained problems followed by CS (in F2 and F4) and FA (in F1 and F3). PSO is not able to converge in F3 and F4 as also shown in Table 4. The complexity for global search increases from F1 to F4 as can be observed in F1, whereas all algorithms successfully converged to global optimum in 200th iteration. However, some algorithm starts to detoriate and converge towards global optimum slower compared to others.

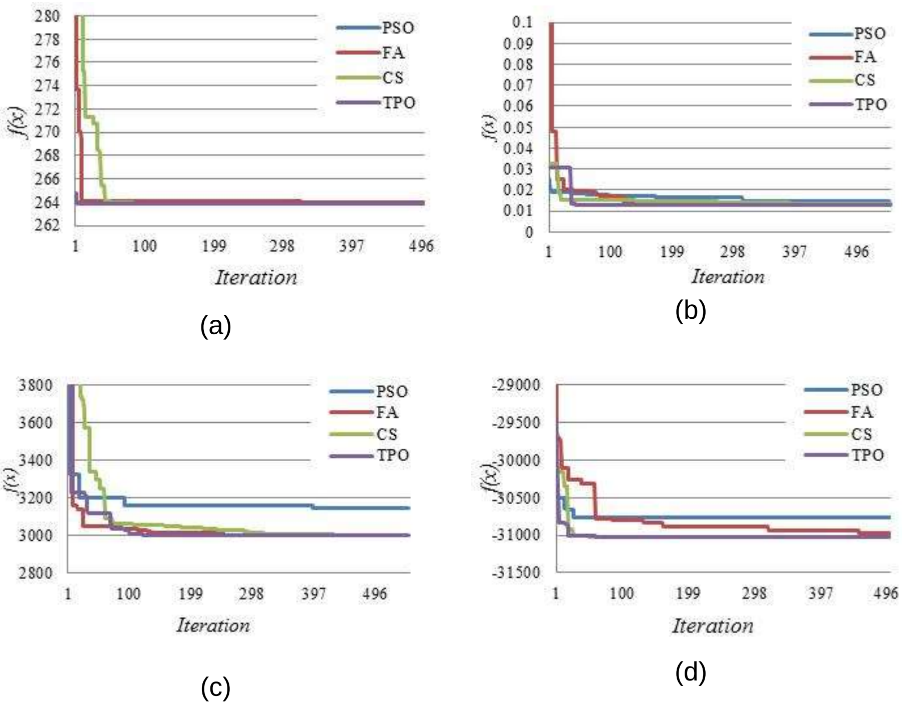


Figure 2. Convergence of best results for each algorithm with (a) three-bar truss, (b) spring design, (c) speed reducer and (d) Himmelblau's nonlinear function

5. Conclusion

This paper proposes a novel Tree Physiology Optimization to solve constrainted optimization problem. The performance of proposed algorithm is compared with other existing algorithms on three engineering design problems and a mathematical nonlinear optimization problem. The performance measures for the comparison include statistical results for each optimization problem and convergence towards global optimum solution. In statistical results, TPO has the best mean value for F1 and F4, PSO has the lowest average for F2 and CS for F3. TPO also has the lowest standard deviation in F1,F2 and F4. The advantage of parallel search of TPO from inidivual leaves and branches as search agents resulted in broader search and thus faster convergence and finding the global optimum. Due to the parallel search (leaves x branches), there is a higher chance to find the current global optimum in each iteration. Therefore the standard deviation of solution in TPO search is smaller. For CS algorithm, the advantage of levy flight led to longer exploration step length in the long run.

With Levy flight, the exploitation in local search is also faster compared to normal random walk. These properties is observed in statistical results in F3 and some other problem types that shows better value compared to several other algorithm. The convergence of CS also show dispersed solution in the beginning of iteration, but then able to exploit the global optimum after some iteration elapsed. FA algorithm has fast convergence in the problem with lower variable dimension. The convergence for higher dimension variables of FA algorithm can be improved further by increasing the number of fireflies. The PSO algorithm has the lowest global optimum in low dimension problem. Nonetheless, its stability problems restrict the success rate as the performance decreases by higher number of dimension variables. The finding imply more similarity studies in diverse fields particularly in real world problem since such problems correspond the the advance of science and technology which may consists numerous constraints that need to be considered. Several paradigms that need be considered are construction engineering, manufacturing and control technology.

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