Performance of Channel Estimation in MIMO-OFDM Systems

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Abstrak

Paper ini mensimulasikan kinerja metode estimasi kanal fading pada orthogonal frequency division multiplexing-multiple input multiple output (OFDM-MIMO), least squares (LS) dan minimum mean squared error (MMSE). Channel impul response (CIR) kanal fading dibutuhkan untuk mengatasi intersymbol interference (ISI). Informasi channel impul response didapat dari estimasi kanal. Iterasi simulasi menggunakan teknik monte-carlo untuk menghasilkan kinerja bit error rate (BER) dan mean squared error (MSE). Hasil simulasi menunjukkan bahwa mean squared error pada sistem MIMO lebih baik dibanding sistem SISO. Dengan estimasi kanal MMSE, sistem MIMO 2Tx-2Rx memberikan perbaikan ±2 dB dibanding sistem SISO untuk nilai MSE 10^{-2}.Selanjutnya, Sistem MIMO 3Tx-2Rx menghasilkan perbaikan 1.5 dB dan MIMO 4Tx-2Rx menghasilkan 3.5 dB. Pada sistem MIMO 2Tx-Rx, teknik estimasi kanal MMSE menghasilkan kinerja lebih baik ±1 dB dibanding teknik estimasi kanal LS untuk untuk MSE 10^{-2} dengan nilai signal to noise ratio (SNR) yang cukup besar. Penyusunan pilot, block type-pilot menghasilkan kinerja lebih baik dibanding comb type-pilot untuk kondisi kanal fading cepat. Block type-pilot memperbaiki sistem ±10 dB dibanding comb type-pilot dengan teknik MMSE untuk nilai BER 2 10^{-2}.

Kata kunci: block-comb type, LS, MIMO-OFDM, MMSE

Abstract

This paper presented the performance of faded channel estimation method on orthogonal frequency division multiplexing-multiple input multiple outputs (OFDM-MIMO) i.e. least squares (LS) and minimum mean squared error (MMSE). Channel impuls response (CIR) was required to overcome the intersymbol interference (ISI). Channel impuls response information was obtained from channel estimation processing. Iterance simulation used monte-carlo technique to determine the performance of bit error rate (BER) and mean squared error (MSE). Simulation results show that the mean squared error performance on MIMO system was better than the SISO system. On MMSE channel estimation, the MIMO 2Tx-2Rx system provided ± 2 dB improvement that compared to SISO system at value of MSE 10^{-2}.Furthermore,MIMO 3Tx-2Rx produce improvement about 1.5 dB, MIMO 4Tx-2Rx improve about 3.5 dB at BER 10-4,respectively.The MIMO 2Tx-2Rx system, MMSE channel estimation produced better performance ± 1 dB than LS channel estimation with sufficient SNR value for MSE 10^{-2} .Pilot arrangement, the simulation results show that the block type-pilot arrangement produced better performance than the comb type-pilot arrangement at fast fading channel. Block type-pilot arrangement system produced better ± 10 dB than the comb type-pilot arrangement with MMSE method at value of BER 2 10^{-2}.

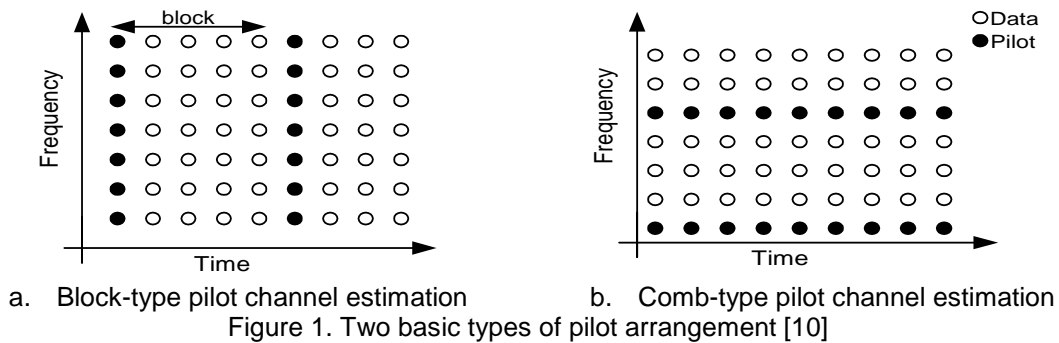*Keywords: block-comb type, LS, MIMO-OFDM, MMSE*

1. Introduction

The combination of orthogonal frequency division multiplexing (OFDM) with multiple input multiple-output (MIMO) and space-time coding has received much attention recently to combat multipath delay spread and increase system capacity. However, to use the advantages that MIMO systems can offer, accurate channel state information (CSI) is required at the transmitter and/or receiver. Channel state information is needed in order to coherently decode the transmitted signal [3].

Due to the multipath channel there is some intersymbol interference (ISI) in the received signal. Therefore a signal detector needs to know channel impulse response (CIR) characteristics to ensure successful equalization (removal of ISI), which can be provided by a separate channel estimator. Usually the channel estimation is based on the known sequence of

bits, which is unique for a certain transmitter and which is repeated in every transmission burst. Thus, the channel estimator is able to estimate CIR for each burst separately by exploiting the known transmitted bits and the corresponding received samples. There are a few different approaches of channel estimation, like Least-squares (LS) or Linear Minimum Mean Squared Error (LMMSE) methods [6].

Optimal placement and energy allocation of training symbols or pilot for both single-carrier and OFDM systems were considered in [7] for frequency-selective block-fading channel estimation. For OFDM systems, the optimal placement of pilot is equal spacing in the frequency domain. In [5], optimal design and placement of pilot symbols for frequency-selective block-fading channel estimation are addressed for single-input single-output (SISO) as well as multiple-input multiple-output (MIMO) [2].

| a. | Block-type pilot channel estimation | b. | Comb-type pilot channel estimation |

Figure 1. Two basic types of pilot arrangement [10]

2. System Model

This paper explains the performance of channel estimation in MIMO-OFDM system. This research explores the MIMO 2 Tx, 3 Tx, and 4 Tx respectively.
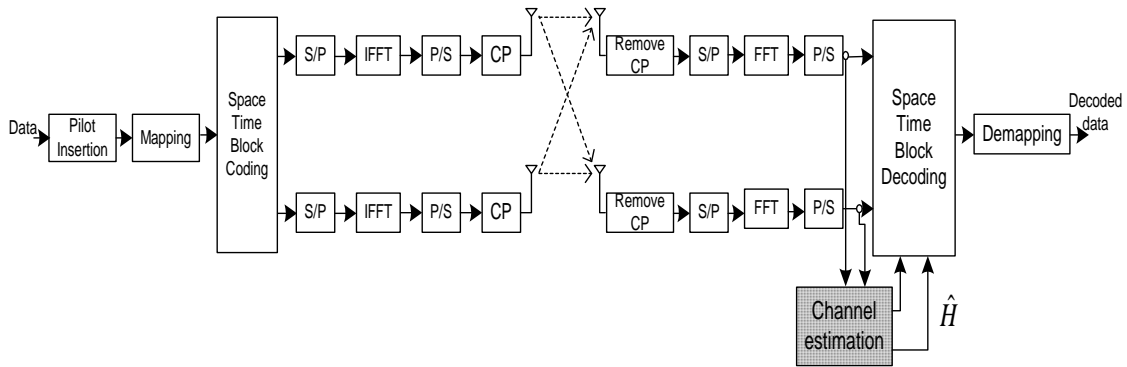

Figure 2. MIMO 2 x 2 system description

The output of the encoder is then split into two ways, one for each antenna as described for the simple case of MIMO space-time coding in [8]. From [8] applied for the OFDM system, we can have the following vectors for antennas 1 and 2 with full rate:

$$X_2 = \begin{bmatrix} X_1 & -X_2^* \\ X_2 & X_1^* \end{bmatrix} \tag{1}$$

$$X_2^1 = \begin{bmatrix} X_1 & -X_2^* & X_3 & -X_4^*, \ldots, X_{N-1} & -X_N^* \end{bmatrix} \tag{2}$$

$$X_2^2 = \begin{bmatrix} X_2 & X_1^* & X_4 & X_3^*, \ldots, X_N & X_{N-1}^* \end{bmatrix} \tag{3}$$

The following complex transmission matrixes of size M_T (number of transmit antennas) = 3 and 4 respectively incorporating a code rate of 1/2.

$$X_3 = \begin{bmatrix} X_1 & -X_2 & -X_3 & -X_4 & X_1^* & -X_2^* & -X_3^* & -X_4^* \\ X_2 & X_1 & X_4 & -X_3 & X_2^* & X_1^* & X_4^* & -X_3^* \\ X_3 & -X_4 & X_1 & X_2 & X_3^* & -X_4^* & X_1^* & X_2^* \end{bmatrix} \tag{4}$$

$$X_4 = \begin{bmatrix} X_1 & -X_2 & -X_3 & -X_4 & X_1^* & -X_2^* & -X_3^* & -X_4^* \\ X_2 & X_1 & X_4 & -X_3 & X_2^* & X_1^* & X_4^* & -X_3^* \\ X_3 & -X_4 & X_1 & X_2 & X_3^* & -X_4^* & X_1^* & X_2^* \\ X_4 & X_3 & -X_2 & X_1 & X_4^* & X_3^* & -X_2^* & X_1^* \end{bmatrix} \tag{5}$$

Table 1. MIMO Tx – Rx Combination

Number of transmit antennas (Tx)	Number of receive antennas(Rx)
2	2
3	2 , 3
4	2,3,4

Minimum mean-square error (MMSE) estimator has good performance but high complexity. Least square [LS] estimator has low complexity, but its performance is not good as that of the MMSE estimator. We use the assumption of a finite length impulse response and QPSK modulation. The performance is presented both in terms of mean-square error (MSE) and bit error rate (BER).

The LS estimator minimizes the parameter $(Y\text{-}\underline{X}H)^H(Y\text{-}\underline{X}H)$ where $(\odot)^H$ means the conjugate transpose operation. It is shown that the LS estimator of $\overline{H}$ is given [10].

$$\hat{H}_{LS} = \underline{X}^{-1}\overline{Y} = \left[\left(X_k \big/ Y_k \right) \right]^T \quad (k=0,\dots,N\text{-}1) \tag{6}$$

The MMSE estimator employs the second-order statistics of the channel conditions to minimize the mean-square error. Denote by $\underline{R}_{gg}, \underline{R}_{HH}$, and $\underline{R}_{yy}$ the auto covariance matrix of $\overline{g}, \overline{H}$, and $\overline{Y}$. $\underline{R}_{gY}$ is the cross covariance matrix between $\overline{g}$ and $\overline{Y}$, also denote by σ_N^2 the noise variance $E\{|\overline{N}|^2\}$. Assume the channel vector $\overline{g}$ and the noise $\overline{N}$ are uncorrelated, it is derived that

$$\underline{R}_{HH} = E\{\overline{H}\,\overline{H}^H\} = E\left\{ \left(\underline{F}_{\overline{g}}\right)\left(\underline{F}_{\overline{g}}\right)^H \right\} = \underline{F}\,\underline{R}_{gg}\,\underline{F}^H \tag{7}$$

$$\underline{R}_{gY} = E\left\{ \overline{g}\,\overline{Y}^H \right\} = E\left\{ \overline{g}\left(\underline{X}\,\underline{F}_{\overline{g}} + \overline{N}\right)^H \right\} = \underline{R}_{gg}\underline{F}^H\,\underline{X}^H \tag{8}$$

$$\underline{R}_{yy} = E\left\{ \overline{Y}\,\overline{Y}^H \right\} = \underline{X}\,\underline{F}\,\underline{R}_{gg}\,\underline{F}^H\,\underline{X}^H + \sigma_N^2\,I_N \tag{9}$$

Assume $\underline{R}_{gg}$ (thus $\underline{R}_{HH}$) and σ_N^2 are known at the receiver in advance, the MMSE estimator of $\overline{g}$ is given by $\hat{g}_{MMSE} = \underline{R}_{gY}\underline{R}_{YY}^{-1}\overline{Y}^{HH}$ [10]. Note that if $\overline{g}$ is not Gaussian, $\hat{g}_{MMSE}$ is not necessarily a minimum mean-square error estimator, but it is still the best linear estimator in the mean-square error sense. At least, it is calculated that

$$\hat{H}_{MMSE} = \underline{F}\hat{g}_{MMSE} = \underline{F}\left[\left(\underline{F}^H\,\underline{X}^H\right)^{-1}\underline{R}_{gg}^{-1}\sigma_N^2 + XF \right]^{-1}\overline{Y} \tag{10}$$

$$= \underline{F}\,\underline{R}_{gg}\left[\left(\underline{F}^H\,\underline{X}^H\,\underline{X}\,\underline{F}\right)^{-1}\sigma_N^2 + R_{gg} \right]\underline{F}^{-1}\hat{H}_{LS} \tag{11}$$

$$= \underline{R}_{HH} \left[\underline{R}_{HH} + \tfrac{2}{N} \left(\underline{X}\, \underline{X}^H \right)^{-1} \right]^{-1} \hat{H}_{LS} \tag{12}$$

The MMSE estimator yields much better performance than LS estimator, especially under the low SNR scenarios. A major drawback of the MMSE estimator is its computational complexity, especially if matrix inversions are needed each time the data in $\underline{X}$ change.

Orthogonality principle is main subject in orthogonal frequency division multiplexing, as follows [1] [9]:

$$\int_a^b \varphi_l(t)\, {}^*_k(t)\,dt = \begin{cases} E_{k,} & \text{if } l=k \\ 0, & \text{if } l \neq k \end{cases}$$

$$= E_k \ (l\text{-}k) \tag{13}$$

$$S_s(kT) = \tfrac{1}{N} \sum_{n=0}^{N-1} A_n\, e^{(j\ n)} e^{j((n\Delta\omega)kT)} \tag{14}$$

OFDM signal equation is equal to discrete fourier transform equation:

$$S_s(k) = \tfrac{1}{N} \sum_{n=0}^{N-1} S(n)\, e^{j\left(\frac{2\pi nk}{N}\right)} \tag{15}$$

In other side, demodulation process uses discrete Fourier transform equation, as follows:

$$S_s(m) = \sum_{k=0}^{N-1} \tfrac{1}{N} S(n) \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi(n-m)k}{N}\right)} \tag{16}$$

$$S_s(m) = \sum_{k=0}^{N-1} S(n)\ (n\text{-}m) \tag{17}$$

Table 2. OFDM parameters

Parameters	Specification
FFT Size	256
Channel	Rayleigh Fading
Pilot type	Block, comb
Constellation	QPSK
GI (Guard interval)	32
Carriers	256
Doppler frequency	80 Hz

3. Results and Analysis

This paper observe both multiplexing diversity gain and spatial diversity gain on MIMO system these are described by both mean squared error and bit error rate performance. This simulation uses monte-carlo method for generating both MSE and BER performances.

3.1. SISO and MIMO Performances

Figure 3 depicts the mean squared error performance of the channel estimator on SISO system, MMSE channel estimator gives a better performance than the LS channel estimator. However, this advantage must be compensated by the complexity of the calculation and knowledge of the channel.

Figure 3 and figure 4 depict the comparison between SISO performance and MIMO 2x 2 performances. MIMO 2x 2 systems produces better performance than SISO system, it is proved on mean squared error performance. MMSE channel estimator on MIMO 2x 2 systems produces about 2 dB better than MMSE channel estimator on SISO system at MSE 10^{-2}. Further, LS channel on MIMO 2x2 system yield about 2.5 dB better than LS channel estimator on SISO system at MSE 10^{-2}. MMSE requires the full a priori knowledge of the MIMO channel

correlations matrix. The better MIMO performance yielded by spatial multiplexing on transmitter system and spatial diversity on receiver system.
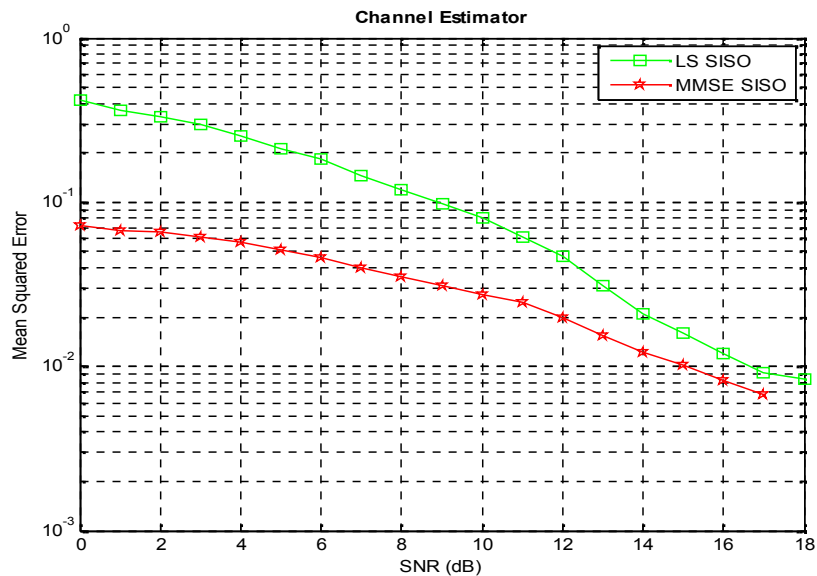

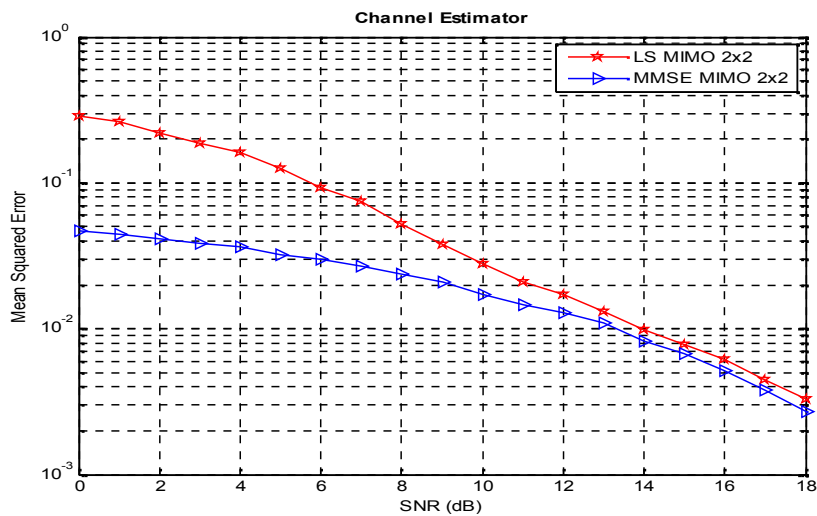Figure 3. MSE performance on single input single output system

Figure 4. MSE performance on MIMO 2 x 2 system

3.2. Performance of Block and Comb Type - Pilot Arrangement

Figure 5 and figure 6 show the BER performance on both block and comb type pilot arrangement on MIMO 2 Tx. Block type – pilot arrangement gives a better performance than the comb type-pilot arrangement. MMSE channel estimator with block type-pilot arrangement produces about 10 dB better than comb type-pilot arrangement at BER 2.0 10^{-2}, the same results also yielded on LS channel estimator system that the block type-pilot arrangement gives the better BER performance than comb type-pilot arrangement. This is due to fast fading that the channel is rapidly changing. The block-type arrangement can compensate the fading.
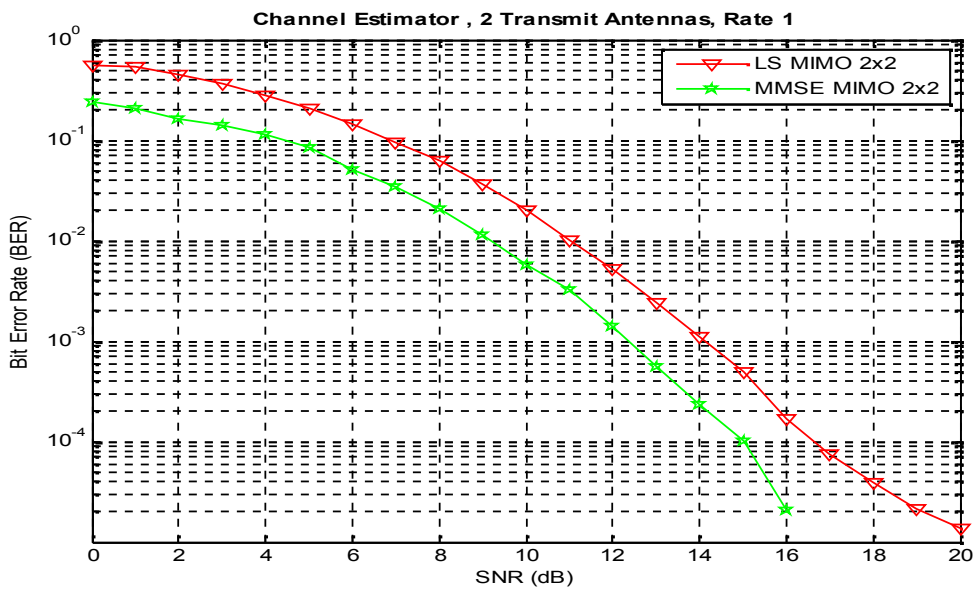
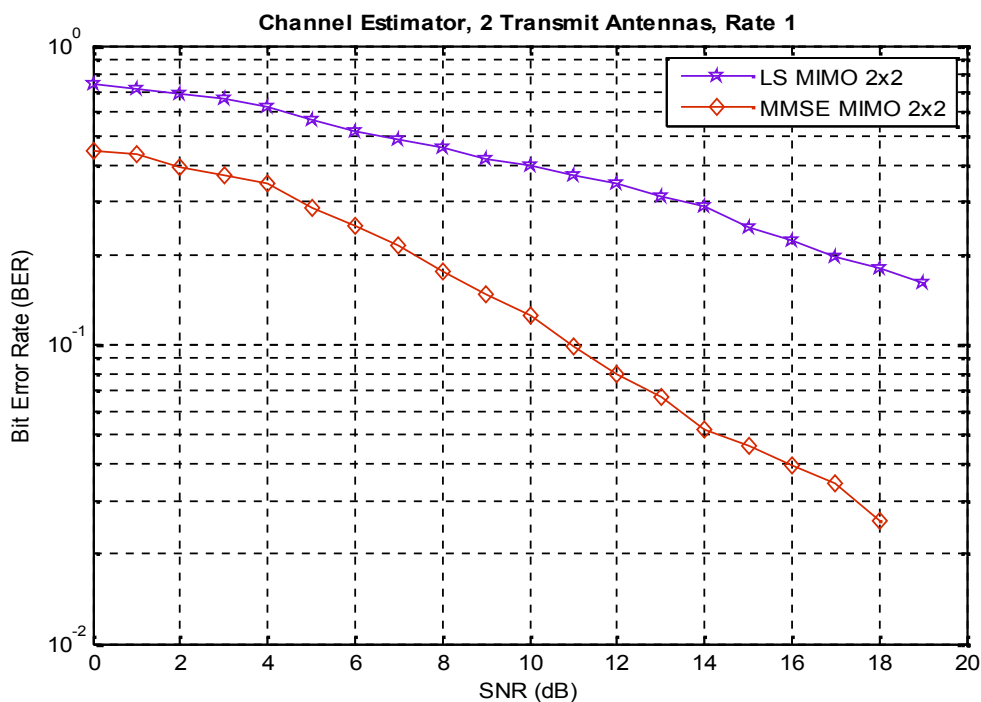Figure 5. BER performance on block type – pilot arrangement with MIMO 2 Tx

Figure 6. BER performance on comb type – pilot arrangement with MIMO 2 Tx

Figure 7 and figure 8 describe the BER performance of MIMO 3 Tx and 4 Tx with combination number of receive antennas (Rx). In this section, we determine using MMSE method and block-type arrangement, these have proven that produce a better performance than another method.
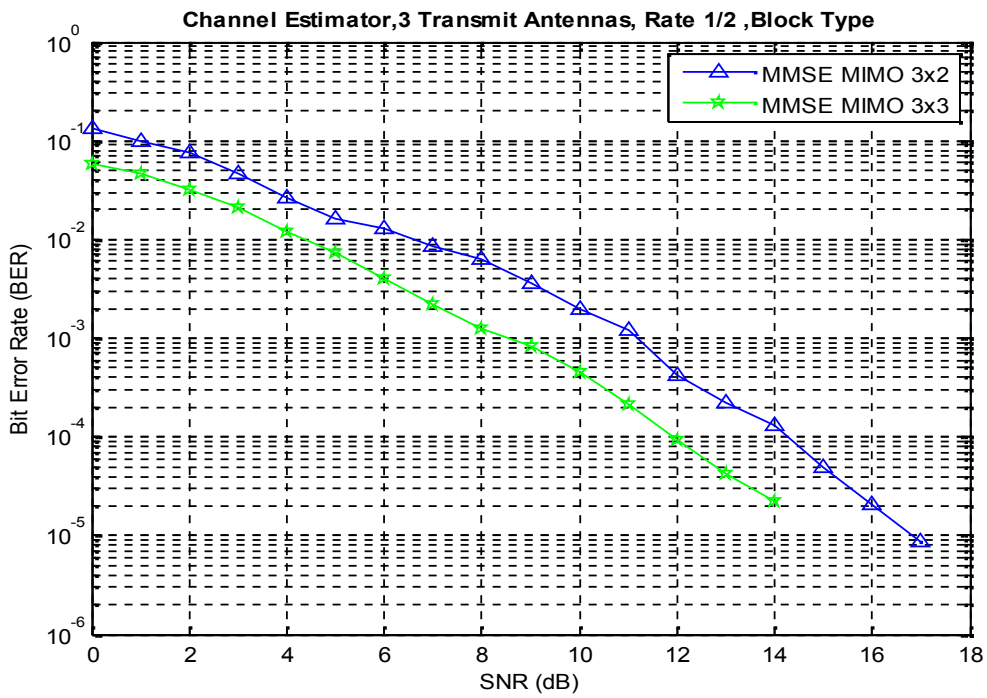
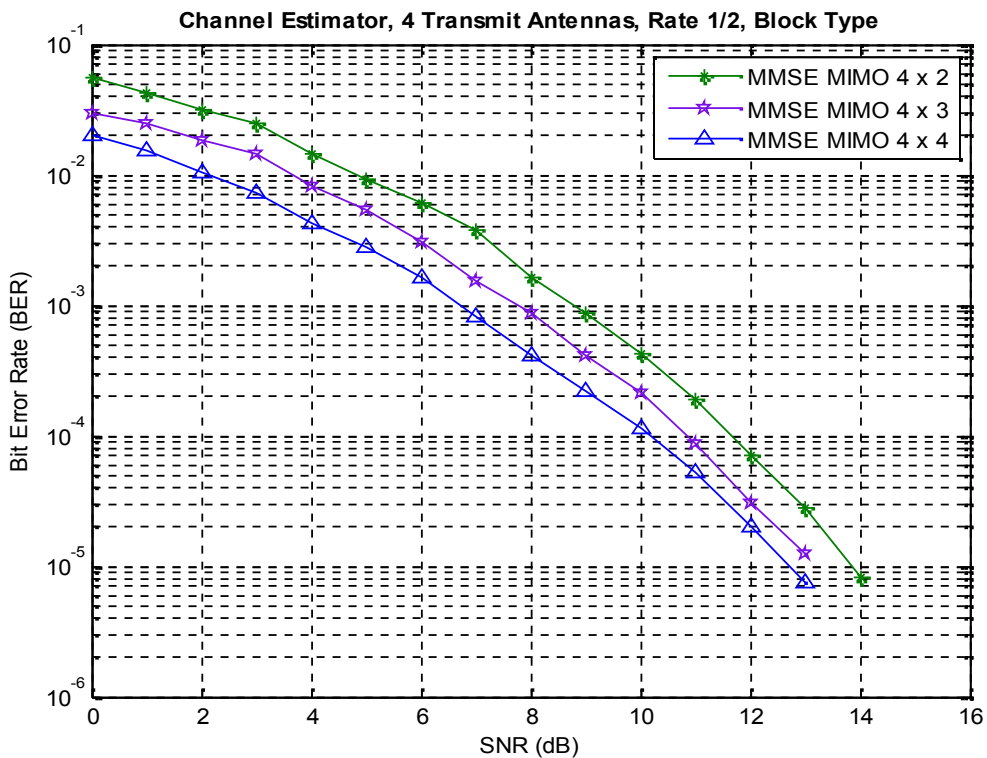Figure 7. BER Performance on MMSE method with MIMO 3 Tx

Figure 8. BER Performance on MMSE method with MIMO 4 Tx

Based on Figure 7 and Figure 8, the MIMO 3 Tx and 4 Tx produce significantly improvement. The MIMO 3Tx – 2Rx yield a better performance about 1.5 dB than MIMO 2Tx-

2Rx at BER 10^{-4}.Furthermore, the MIMO 4Tx-2Tx produces the best performance. The MIMO 4Tx-2Rx gives advance about 3.5 dB than 2Tx-2Rx at BER 10^{-4}. Higher number transmit antennas produce a bigger spatial multiplexing gain. Furthermore, higher number of receive antennas give a higher spatial diversity gain.

4. Conclusion

MMSE channel estimator produces a better performance than LS channel estimator. However, the complexity of the calculation must be considered. Relying on both spatial multiplexing gain and spatial diversity gain, the MIMO system yields a better performance than the SISO system. The last point, block type-pilot arrangement produces a better BER performance than comb type-pilot arrangement on fast fading channel.

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