

# Radial Derivative and Radial Inversion for Interpreting 4D Gravity Anomaly Due to Fluids Injection Around Reservoir

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## Abstract

The 4D gravity or time lapse gravity has been used many reseracher to identify fluid injection in oil reservoir. The objective of this study is to find the better way in interpreting 4D gravity anomaly due to fluid injection around the reservoir. Radial Derivatives are derivative values of gravity anomalies against horizontal distances in the radial direction. Radial inversion is a two-dimensional inversion of lines with radial directions resulting in a 3-dimension model. Time lapse microgravity research have been performed in "X Oil Field" with amount of 604 data point covering area of 4000 m x 5000 m. This Radial derivative and Radial inversion have been aplied at an injection well of the X oil field. The yield show that 4D gravity anomaly value due to injection is 0.02 mGal to 0.36 mGal. Radial derivative value in the area is 0 micro Gal/cm to 0,012 mGal/meter. Radial inversion shows radius of fluid front movement is 304 meters to 1120 meters. Radial derivative and Radial inversion have been proven fairly good to identify injected fluid movement in the reservoir.

**Keywords:** radial, derivative, inversion, 4D gravity, reservoir

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## 1. Introduction

The 4D micro-gravity method or well known as time lapse microgravity is the development of gravity method with time as the fourth dimension. This method is characterized by repeatable measurements either daily, weekly, monthly or yearly using very high-accuracy gravity measurements supported by positioning and height measurements with high accuracy [1]. The 4D gravity method has been widely used for the identification of subsurface changes. Eiken et al, using intermittent micro gravity, to monitor gas production in subsea reservoirs with a gross gravimeter of up to 4 micro Gals [2]. Gettings et al, measured the value of 4D micro gravity around geothermal geysers sources to detect subsidence [3].

Akasaka and Nakanishi separated the gravity anomaly of geothermal sources from the effect of groundwater change [4]. Rahman et al. succeeded in monitoring fluid injection in reservoirs using the 4D gravity method [5]. Dafis et al., measured the 4D gravity anomaly to monitor the rate of water injection in artificial aquifer storage and recovery (ASR) aquifers in Leyden Colorado [6]. Sarkowi examines the relationship between ground water changes with vertical gravity gradient changes in the city of Semarang and the surrounding areas [7].

Tsuboi dan Kato, examine the properties of the second derivative of gravity data based on the Laplace equation [8]. Reilly, studies formulations for differentiation, smoothing and interpolation of gravity data [9]. Abdelrahman dkk. [10], using derivatives analytically to detect fault zones [10]. Oruc, detect boundaries of anomaly source and estimate depth using horizontal gravity gradient data. This method is performed in the Kozakli-Central area, Anatolia, Turkey [11]. Ansari, proposed a new method for detecting anomaly edges with Analitic Signal of Tilt Angle (ASTA). This method has been tested on magnetic and gravity data in southwestern England [12]. Aydogan, extracting lines of gravity anomalies and performing horizontal gradient calculations for interpretation of geological structure boundaries. This method has been tested

in the Tuzgolou basin, Anatolia, Turkey [13]. Tatchtum, make geological tilt models in various shapes, the anomaly respon and differentiated responses horizontally. This method was tested in the Foubnon fault zone in Cameroon [14]. Chijun, measure vertical derivatives directly to get a more accurate geoid value up to the order in centimeter level [15]. Aku, using vertical derivative analysis to determine lithological vertical boundaries to sharpen the analysis of gravitational interpretation in Gusau, Nigeria [16]. Askari, using the slope angle of the total normalized derivative of the gravity anomaly to detect geological structural boundaries. This method has been performed in the Sea of Oman, southeast of Iran [17]. Wahyudi, use vertical derivatives from 3D gravity data for interpretation of geological structures. This method has been tested in a very wide area, namely Central Java, East Java and Madura [18]. Pasteka dkk., using derivative transformations from Bouguer anomalies for geological interpretation. This method test was carried out in a very large area in Slovakia [19].

The world's oil demand is always increasing over time, while oil availability is running low. Efforts to fulfill oil demand are performed by exploring new oil resources and optimizing on producing oil reservoirs wich also well known as Enhancement Oil Recovery (EOR). One way to increase production is injecting fluids into the reservoir. This fluid will move away from the injection well and push the oil toward the production well. In the fluid injection process, the density change of the pore fluid mass will result in a 4D gravity anomaly. Time lapse gravity anomaly due to injection is the gravity value difference of post-injection with pre- injection. The reservoir model and the 4D gravity response due to the fluids injection is shown in Figure 1. The 4D anomaly is positive because the density of injected fluid (water) is graeter than the oil filling the pore.

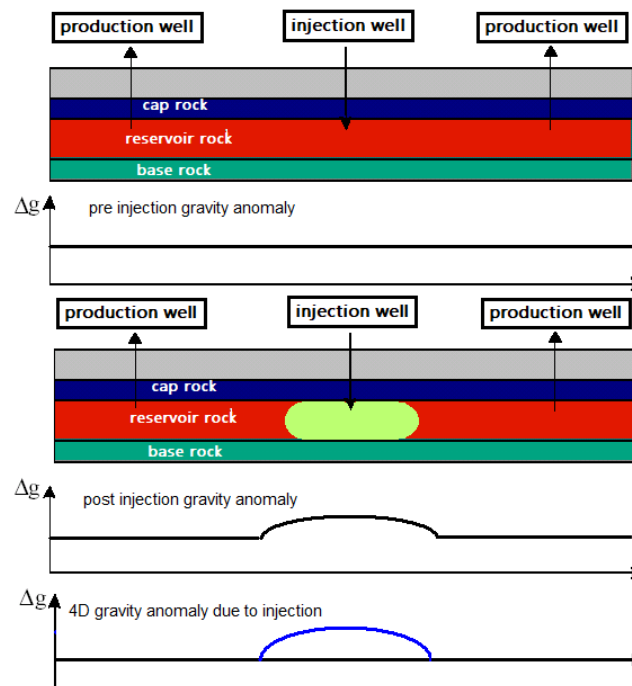


Figure 1. Reservoir model and the 4D gravity response due to fluids injection in reservoir

The identification of fluid injection in reservoir using gravity method has been conducted by many researchers. The use of this method is divided into two major parts of the method of gravity on the surface and gravity of the well hole. Vasilevskiy et al used the gravity data of the well-hole by interpretation through inversion [20]. Sarkowi made the interpretation of surface gravity with the forward model [21], while Zuhdi et al made the same way by performing radial derivative as shown in Figure 2 (a) [22]. The main problem of 4D gravity in identification of fluid injection in reservoir is the difficulties to identify contact of injected fluid and pre-injection

fluid. To solve this main problem authors propose radial derivatives and radial inversion. The objective of this research is to construct computer program of radial derivative and radial inversion of 4D gravity anomaly and show the ability to show boundary of density contrast in injected reservoir. The data was taken with Lacoste Romberg type 504 and Scintrex C 63, consist of 604 measurement point covering area of 4000 mx5000 m, in 3 periods ie. January, May and September in 2003. Radial derivative and radial inversion have been performed in the part of the area covering 1000m x 1000m with an injection well as the center.

## 2. Research Method

### 2.1. Time Lapse Micro Gravity Method

According to Kadir, a 3-dimensional object with a mass density distribution  $\rho(\alpha, \beta, \gamma)$ , gives the effect of gravity at point  $P(x, y, z)$  on the surface within a given time interval ( $\Delta t$ ) given by [23]:

$$\Delta g(x, y, z, \Delta t) = K \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\Delta \rho(\alpha, \beta, \gamma, \Delta t)(z - \gamma)}{[(x - \alpha)^2 + (y - \beta)^2 + (z - \gamma)^2]^{3/2}} d\alpha d\beta d\gamma \quad (1)$$

In (1) if the change of gravity anomaly only depends on density change (no geometrical change), then it can be approximated by:

$$\Delta g(x, y, z, \Delta t) \cong \kappa \cdot \Delta \rho(x, y, z, \Delta t) \quad (2)$$

where  $\kappa$  is a constant corresponding to the geometry and volume of anomaly source objects, whereas:

$$\Delta g(x, y, z, \Delta t) = g(x, y, z, t') - g(x, y, z, t) \quad (3)$$

$\Delta g(x, y, z, \Delta t)$  is a gravity change related to density changes due to fluid injection in reservoir. If pore volume is filled with oil at time  $t$ , and there is a replacement of oil with injected fluid at time  $t'$ , then the density change is given by the (4) [24]:

$$\Delta \rho = \rho' - \rho = \Phi (\rho_f - \rho_o) (S_o - 1) \quad (4)$$

where  $\rho_f$  is density of injected fluid and  $\rho_o$  is density of oil. Oil saturation  $S_o$  is defined as the ratio of volume  $V_m$  to the overall pore volume of  $V_p$ , and  $\Phi$  is the total porosity.

$$\Delta g = k \Phi (\rho_f - \rho_o) (S_o - 1) \quad (5)$$

by replacing  $S_o$  with  $V_o / V_p$ , (5) becomes:

$$\Delta g = \Phi (\rho_f - \rho_o) \{(V_o / V_p) - 1\} \quad (6)$$

the anomaly  $\Delta g$  will be positive because the oil (low density) is replaced with water (higher density). If the injected fluid has bigger density, its time lapse anomaly will be positive.

### 2.2. Radial Derivative

Radial derivative is a derivative of the gravity anomaly value to the horizontal distance with the radial direction of a particular point. The horizontal derivative of a gravitational anomaly is the derivative of the gravity value in the direction of a particular straight line. If the straightline cuts perpendicular to a contrast density boundary, the derivative value will be of great value. For these purposes radial derivatives are made. The radial derivative center point is chosen based on the center of anomaly.

Radial derivatives will be easier to perform in vertical cylindrical coordinates. The gravity anomaly value  $\Delta g_z(x, y, z)$  caused by the density anomaly  $\Delta \rho(\alpha, \beta, \gamma)$  is written as:



### 2.3. Radial Inversion

The 4D gravity anomaly of the reservoir contaminated with shallow effect anomaly is then separated by filtering. The residual (local effect) of the filtering is an undesirable shallow anomaly, while the regional one is the anomaly part of the reservoir which is then be inverted by inversion program. Figure 2 (b) shows some 2D inversion lines that produce a radial inversion. The radial inversion performed in this case is assumed to be an anomaly derived only from the reservoir. This inversion is based on the inversion of a cylindrical-like reservoir shape with thickness corresponding to the reservoir thickness. The inversion result is expected to illustrate the actual reservoir geometry shape, which includes density contrast and the fluid movement radius in the reservoir.

Inversion theory can be regarded as an art to obtain various physical parameters with sufficient knowledge about forward modeling [26]. Relation of observed data and model can be written as:

$$d = g(m) \quad (12)$$

where  $d$  is the observed data,  $m$  is the model parameter and  $g$  is forward function which is the prediction function of the model known as Kernel function. If the observed data are  $N$  and the model parameter of  $M$  can be related to the Kernel's matrix by the (13):

$$d = G m \quad (13)$$

where  $d$  is a  $N \times 1$  matrix,  $G$  is a  $N \times M$  Kernel matrix and  $m$  is a model parameter of  $M \times 1$  size. The inversion relationship of model parameters with observed data can be written as:

$$m = [G^T G]^{-1} G^T d \quad (14)$$

The measured gravity value actually has a non-linear relationship to the parameters of the model in search of density contrast, radii and position, but this relationship can be solved by linear approximation. The principle of this method is the LSQR with special development. The formulation of non-linear inversion method with approach linear principle using the first Taylor series and ignoring the higher rates of the Taylor series around  $x + \Delta x$  as follows:

$$f_{(x+\Delta x)} = f_{(x)} + f'_{(x)} \Delta x \quad (15)$$

where  $f_{(x+\Delta x)}$  is the measurement data written with  $d$ ,  $f_{(x)}$  is the result of the model response ie.  $g(m_0)$  and  $f'_{(x)}$  is first derivative of the model parameter forming the Jacobi matrix  $J_0$ , so:

$$d = g(m_0) + J_0 \Delta m_0 \quad (16)$$

The Jacob's matrix consists of:

$$J_A = \left[ \frac{\partial g_i}{\partial m_A} \right], J_d = \left[ \frac{\partial g_i}{\partial m_d} \right] \text{ and } J_r = \left[ \frac{\partial g_i}{\partial m_r} \right]$$

where  $A$  is the Amplitude factor  $d$  is the depth and  $r$  is the radius of the fluid injection front, so the Jacobi matrix of the cylinder model can be written as:

$$J_0 = [J_A \ J_d \ J_r]$$

The Kernel model of this radial derivative function is based on Finite horizontal slab which can be written as:

$$g = 2G\rho t \left\{ \pi + \tan^{-1} \left( \frac{x+0.5L}{z} \right) + \tan^{-1} \left( \frac{0.5L-x}{z} \right) \right\} \quad (17)$$

from (16), we can obtain:

$$d - g(m_0) = J_0 \Delta m \quad (18)$$

so,

$$\Delta m = [J_0^T J_0]^{-1} J_0^T (d - g(m_0)) \quad (19)$$

with  $J_0^T$  is the transpose of matrix  $J_0$ , since

$$d - g(m_0) = \Delta d \tag{20}$$

then  $\Delta d$  has a linear relationship with  $\Delta m$  so that it can be solved by LSQR method.

The principle of linear inversion solution is a recurring count that will yield the value  $m = m_0 + \Delta m$  so that the minimum  $\Delta d$  value is obtained. With the RMS value of  $\Delta d$  small enough to some extent, iteration is then stopped. With the very small value of  $\Delta d$  then the value of  $m \approx m_0$  so that the model parameters very close to the real value [26]. The flowchart of the process is shown in Figure 3.

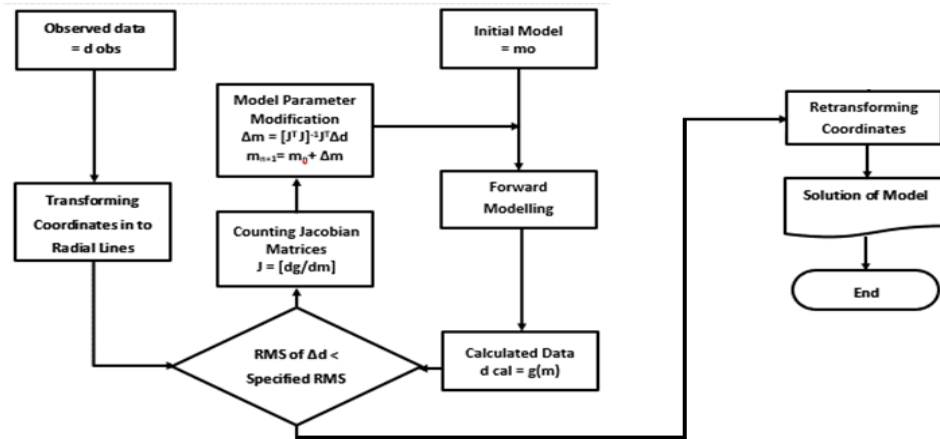


Figure 3. Flow chart in the radial inversion programming

### 3. Results and Analysis

#### 3.1. Radial Derivatives for Injection Fluid Identification

An anomaly due to injection that has been removed from shallow effect is then treated with radial derivative. This treatment is intended to find the front of fluid injection. The front of fluid injection made a density contrast which result measurable anomaly of gravity on the surface. Figure 4 shows the radial derivative of 4D gravity anomaly due to injection from January to May. The middle of the blue zone on the image can be interpreted as the contact boundary between the injection fluid and the previous pore fluid. The yield show maximum radial derivative in the area is 0,012 mGal/meter and the minimum value of 0 micro Gal/m.

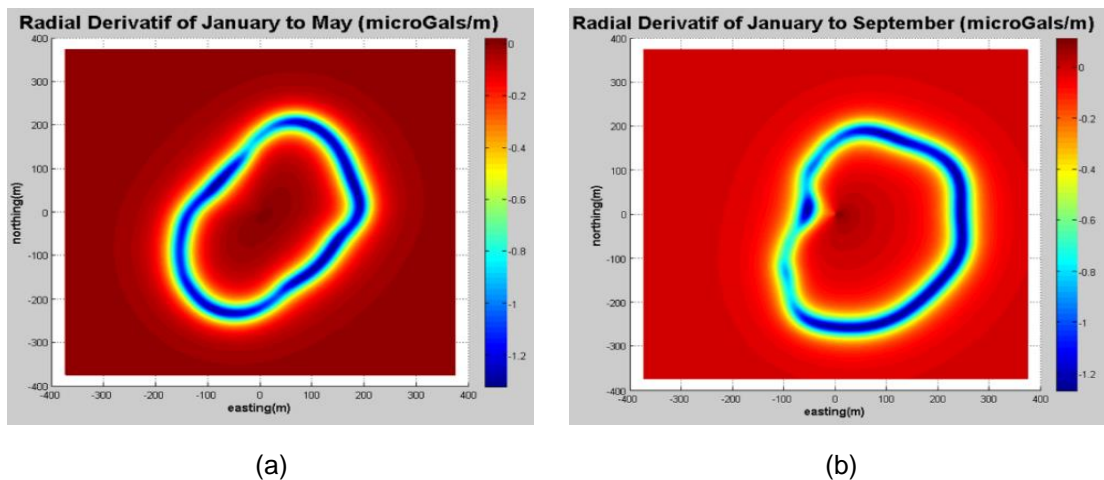
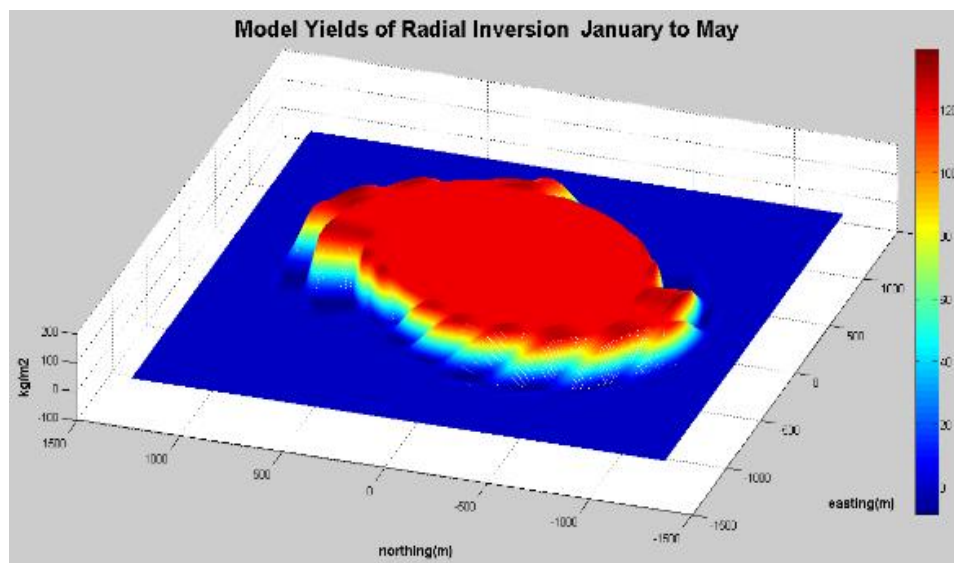


Figure 4. Radial derivative of 4D gravity anomaly due to injection: (a) January to May period, (b) January to September period

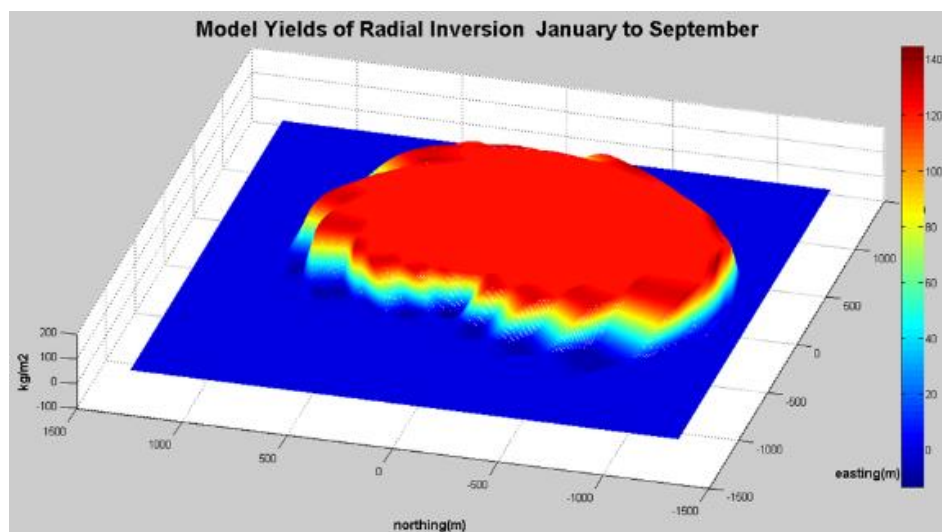
### 3.2. Radial Inversion For identification of Injection Fluid

The input of this Radial Inversion program is a file of 4D gravity anomaly containing position (x, y) and gravity values in regular grid. The data is then extracted by the number of specific lines to get the inversion lines. Each inversion line is then performed inversion process with kernel function as finite horizontal slab (17). The yields of the inversion parameter are the amplitude factor and the radius of fluid injection front. The result of this inversion data is then unified and re-gridded to obtain 3-dimensional model of reservoir injection fluid.

Figure 5 shows the fluid injection volume model in the reservoir based on the radial inversion method. The flat blue zone is interpreted as uncontaminated reservoir and the red zone is volume of the injection fluid infiltrated the reservoir. The volume of the injection fluid has a shape tends to be straight to the radii because the kernel matrix of the program only allows us to obtain a uniform volumes thickness value toward the radius. Reservoir model due to fluid injection as a result of radial inversion has maximum value of 1120 meter and minimum value of 304 meter.



(a)



(b)

Figure 5. Injection fluid volume model on the reservoir based on radial inversion results:  
(a) January to May period, (b) January to September period

#### 4. Conclusion

Radial derivatives and Radial Inversion has been proven fairly good for identification of fluid injection in the reservoir. This method is useful to help more accurate analysis of 4D gravity around the reservoir. Radial derivatives are not only applicable to 4D anomalies due to fluid injection in the reservoir, but also effective for the analysis of ordinary gravity anomalies with horizontal slab shape. Radial Derivative Methods can also be applied to ordinary gravity anomalies and become tools for further interpretation of existing gravity data.

Radial inversions can be applied to time lapse gravity data around injection wells for identification of additional mass (positive density contrast) or mass reduction (negative density contrast). Radial inversion can also be used in an ordinary gravity anomaly (3D) to detect boundary of horizontal slab. Radial Inversion is fairly good to provide a physical reservoir parameters. This program is useful for general interpretation of gravity and can be a reliable additional software as a new way of interpretation.

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